ANALYSIS AND CONTROL OF TORQUE HARMONICS OF AN INDUCTION MOTOR FED BY A THREE PHASE CURRENT SOURCE INVERTER

By

SANJAY AGARWAL

DEPARTMENT OF ELECTRICAL ENGINEERING

DIAN INSTITUTE OF TECHNOLOGY KANPUR

JULY, 1984

ANALYSIS AND CONTROL OF TORQUE HARMONICS OF AN INDUCTION MOTOR FED BY A THREE PHASE CURRENT SOURCE INVERTER

A Thesis Submitted
in Partial Fulfilment of the Requirements
for the Degree of

MASTER OF TECHNOLOGY

By

SANJAY AGARWAL

to the

DEPARTMENT OF ELECTRICAL ENGINEERING

INDIAN INSTITUTE OF TECHNOLOGY KANPUR

JULY, 1984

DEDICATED TO

MY FATHER

whose active guidance and constant
encouragement has shown me the direction;

MY MOTHER

whose blessings saved me from many
pitfalls and led me on the present path.

CERTIFICATE

Certified that this work entitled, 'ANALYSIS AND CONTROL OF TORQUE HARMONICS OF AN INDUCTION MOTOR FED BY A THREE PHASE CURRENT SOURCE INVERTER' by Sanjay Agarwal is carried out under my supervision and this has not been submitted elsewhere for a degree.

Dr. Avinash Joshi
Assistant Professor
Department of Electrical Engineering
Indian Institute of Technology
Kanpur.

ACKNOWLEDGEMENTS

I express my deep sense of gratitude to Dr. Avinash Joshi for suggesting this topic and actively participating at each level with invaluable guidance and encouragement without which this work would not have been possible.

I take this opportunity to thank my brother, Manoj Agarwal, for his association in the preparation of the thesis. Many thanks are due to my friends, whose assistance has consisted not so much of direct contribution to the work as to the morale-boosting encouragement.

Sanjay Agarwal

(SANJAY AGARWAL)

TABLE OF CONTENTS

LIST OF SYMBOLS

I_{dc}	DC link current
M	Mutual inductance between stator and rotor phase referred to the stator
ω_r	Angular velocity of the rotor, elec. rads/sec.
p	The operator, (d/dt)
p^2	The operator, (d^2/dt^2)
a	Inverse of motor time constant
T_q	Instantaneous torque of the motor, N-m
T_c	$(\pi/3\omega)$, Duration of the interval of the inverter perioc
t_c	Total commutation time
i_{d1}	Instantaneous stator current along d axis
i_{q1}	Instantaneous stator current along q axis
i_{d2}	Instantaneous rotor current along d axis referred to the stator
i_{q2}	Instantaneous rotor current along q axis referred to the stator
X_1	Variable proportional to instantaneous value of the flux linkages along q axis
X_2	Variable proportional to the instantaneous value of the flux linkage along q axis
I_{do}	Initial value of the rotor current of an interval along d axes referred to stator
I_{qo}	Initial value of the rotor current of an interval along q axis referred to stator
	Refers to the change in the variable at the point of consideration
β	Inverse of the time constant of the modulating exponential waveform

Frequency of the modulating cosinusoidal waveform, rads/sec

nth harmonic of the torque

Slope of the isocline

Torque constant, [M * (Phase/2) * (Pole/2)]

a phase current of the stator

Variable inversely proportional to 'a' phase current

Slope of variable y, (dy/dt)

Inverter frequency, rads/sec

Initial value of the variable X_2 in an interval

Initial value of the variable X_1 in an interval

ABSTRACT

The present work deals with the steady state analysis and control of torque harmonics of the induction motor fed by three phase current source inverter.

The analytical solutions for the rotor currents, stator voltages and torque waveform in the stationary dq frame attached to the stator have been obtained. The rotor currents and torque harmonic spectrums have also been obtained through frequency domain analysis.

The next part of the thesis deals with the control of the torque harmonics by modulation of dc input current. A detailed study has been made of the modulation by exponential and cosinusoidal waveforms and their effects on various torque harmonics. A general nonlinear second order equation has been derived for obtaining the waveshape of the inverter input current required to produce a desired torque waveform. A general analytical solution of this equation has been done for the case of stationary rotor. Further for the case of rotating rotor and constant torque, this equation has been solved using phase plane analysis and numerical integration. The current profile for producing constant torque at any rotor speed in the motoring mode has been obtained.

CHAPTER 1

INTRODUCTION

It is well known that an induction motor supplied by a current source inverter draws line currents with an approximately quasi square waveform [1]. Consequently, the general analysis of the motor with sinusoidal voltages and currents is not directly applicable in this case. The most popular approaches for the analysis of ac motor drives, which employ non sinusoidal sources are harmonic superposition techniques using either symmetrical components [2] or method of multiple reference frames [3]. The analysis of the performance of the induction machine fed by a current source inverter has been carried out in [1],[4],[5],[6],[7] using dq axes model of induction motor. Several methods of solving dq equations have been proposed. In [4],[5],[6], the analysis of induction motor is carried out assuming ideal waveshape for stator current waveform. In [7] the analysis has been done using digital simulation. None of these give direct analytical expressions for calculating referred rotor currents stator voltages and torque waveform. In this thesis analytical expressions are obtained using d-q model of the machine for referred rotor currents, stator voltages and electromagnetic torque. Analysis has been done for both the cases of ideal current source inverter and the inverter with nonzero commutation time. For the latter,

therise and fall of the stator current during commutation has been taken of cosinusoidal nature [1].

It is possible to control the torque harmonics of an induction motor fed by a current source inverter either by modulating the dc current input of the inverter or by modulation within the inverter. In [8], [9] the effects on the torque harmonics by modulating the inverter input current by a particular exponential waveform has been studied. Modulation within the inverter is referred to as pulse width modulation in literature [10]. Reference [10] gives the effects on torque harmonics because of the pulse width modulated current source inverters. In this thesis a detailed study has been made of the modulation of inverter input current with exponential and cosinusoidal waveforms. It has been shown that the parameters of the modulating waveforms control the torque harmonics. Optimum parameters for eliminating the dominant sixth harmonic have been calculated. A general nonlinear second order equation has also been derived for obtaining the waveshape of the inverter input current required to produce a desired torque waveform. A general analytical solution of this equation has been done for the case of stationary rotor. Further for the case of rotating rotor and constant torque this equation has been solved using phase plane analysis and numerical integration.

The dq model of the induction motor, with axes attached to the stator has been used throughout for the analysis. A brief description of dq axes model for three phase induction motor is given in Sec. 1.1.

In Chapter 2, the steady state analysis of three phase induction motor fed by an ideal three phase current source inverter has been done to compute the referred rotor current and the stator voltage waveforms. The analytical solutions are obtained for rotor currents and stator voltage for three intervals of the inverter period. Boundary conditions are verified analytically for the case of stationary rotor. The resulting expressions for the case of the rotating rotor are complicated and hence, closed form equations have not been obtained. A computer program has been written to compute the rotor currents. Analytical solution results in a considerable reduction of computer time compared to the numerical solution of dq equations. The boundary conditions for the case of rotating rotor are verified with the computer program. The values of the rotor current harmonics have also been obtained through frequency domain analysis of the induction motor. It has been shown that the rotor harmonics as obtained from the analytical solution are in close agreement to those obtained through frequency domain analysis.

In Chapter 3, the analysis similar to that in Chapter 2 has been done. In this case the inverter is assumed to have

a nonzero commutation time. During this commutation time, the rise and fall of the current is approximated by a consinusoidal function [1].

In Chapter 4. the electromagnetic torque of the current fed induction motor has been computed. Two methods to compute the torque have been given. One obtains the analytical solution of torque using the time domain expressions of stator and referred rotor currents in dq frame. The other computes the torque harmonic spectrum using the harmonic components of the stator and rotor currents in d-q frame. Using these procedures. the torque waveform and spectrum has been studied for the case of ideal current source inverter supplying an induction motor, to the modulation of the inverter input dc current by exponential modulation.

In Chapter 5, a detailed study has been made of the effects on torque harmonics due to the modulation of the inverter input dc current by exponential and cosinusoidal waveforms. The exponential modulation for stationary rotor case gives constant torque. Cosinusoidal type of waveform may be present in the inverter input current due to imperfect filtering in the dc link. It has been shown that it is not possible to get an exactly constant torque with these types of modulation for the case of rotating rotor. However, by a certain choice of the parameters of the modulating waveforms, it is possible to reduce certain torque harmonic.

Chapter 6 deals with the calculation of stator current profile of the induction motor fed by current source inverter to produce a specified torque waveform. A general nonlinear second order equation has been derived for obtaining the waveshape of the inverter input current required to produce the desired torque waveform. A general analytical solution of this equation has been done for the case of stationary rotor. Further for the case of rotating rotor and constant torque this equation has been solved using phase plane analysis and numerical integration. The current profile for obtaining a constant torque at any rotor speed in the motoring mode has been obtained.

1.1 DQ MODEL FOR INDUCTION MOTOR

The development of the dq model for the induction motor using arbitrary revolving frame has been done by Krause [3]. In this section the equations of motor performance and the transformations from three phase balanced system to the dq frame have been outlined briefly using [3].

Fig. 1.1 shows the angular relation of the stator and rotor axis of a three phase machine with the third set which is an orthogonal set (dq axis) rotating at an arbitary electrical angular speed $d\Theta/dt$. It is clear that a_1-b_1-c_1 set is fixed in the stator and a_2-b_2-c_2 set is fixed in rotor

and hence rotates at an angular velocity ω_r. Subscript 1 is used for stator quantities and subscript 2 for rotor quantities. The time zero angular relationship between three set of axes can be selected arbitarily; however, it is convenient to assume that at time zero, a_1 and 'd' axes coincide.

The transformation equations which for the balanced systc can be correlated to the angular relation of the axes shown in Fig. 1.1 are written as follows :

$$f_{d1} = 2/3\ [f_{a1} \cos\theta + f_{b1} \cos(\theta - \tfrac{2\pi}{3}) + f_{c1} \cos(\theta + \tfrac{2\pi}{3})] \quad (1.1)$$

$$f_{q1} = 2/3\ [-f_{a1} \sin\theta - f_{b1} \sin(\theta - \tfrac{2\pi}{3}) - f_{c1} \sin(\theta + \tfrac{2\pi}{3})] \quad (1.2)$$

$$f_{d2} = 2/3\ [f_{a2} \cos\beta + f_{b2} \cos(\beta - \tfrac{2\pi}{3}) + f_{c2} \cos(\beta + \tfrac{2\pi}{3})] \quad (1.3)$$

$$f_{q2} = 2/3\ [-f_{a2} \sin\beta - f_{b2} \sin(\beta - \tfrac{2\pi}{3}) - f_{c2} \sin(\beta + \tfrac{2\pi}{3})] \quad (1.4)$$

where

$$\beta = \theta - \theta_r \quad (1.5)$$

In these equations the variable f can represent either voltage, current or flux linkage. The equations are restricted in that the instantaneous angular displacement θ of the arbitary reference frame must be continuous finite function.

It has been shown in [4] that the equations of motor performance for squirrel cage induction motor for a stationary

reference frame are

$$\begin{bmatrix} 0 \\ 0 \end{bmatrix} = \begin{bmatrix} Mp & \omega_r M & (r_2+L_{22}p) & \omega_r L_{22} \\ -\omega_r M & Mp & -\omega_r L_{22} & (r_2+L_{22}p \end{bmatrix} \begin{bmatrix} i_{d1} \\ i_{q1} \\ i_{d2} \\ i_{q2} \end{bmatrix} \tag{1.6}$$

$$\begin{bmatrix} V_{d1} \\ V_{q1} \end{bmatrix} = \begin{bmatrix} (r_1+L_{11}p) & 0 & Mp & 0 \\ 0 & (r_1+L_{11}p) & 0 & Mp \end{bmatrix} \begin{bmatrix} i_{d1} \\ i_{q1} \\ i_{d2} \\ i_{q2} \end{bmatrix} \tag{1.7}$$

$$T_q = M(m/2)\ (P/2)\ (i_{q1}\ i_{d2} - i_{d1}\ i_{q2}) \tag{1.8}$$

where m is the number of phases, P is the number of poles, L_{11} is the leakage inductance of the rotor, L_{22} the leakage inductance of the rotor referred to the stator, M is mutual inductance and 'p' refers to 'd/dt'. In these 'T_q' is the instantaneous torque of the motor.

In order to solve the above dq equations, the three phase input excitation currents are transformed into two phase dq currents. The transformation connecting 3 phase to 2 phase variables with assumed direction of coordinates as shown in Fig. 1.1, for the case of balanced stator current gives

$$i_{d1} = i_{a1} \tag{1.9}$$

$$i_{q1} = \frac{1}{\sqrt{3}}\ (i_{b1} - i_{c1}) \tag{1.10}$$

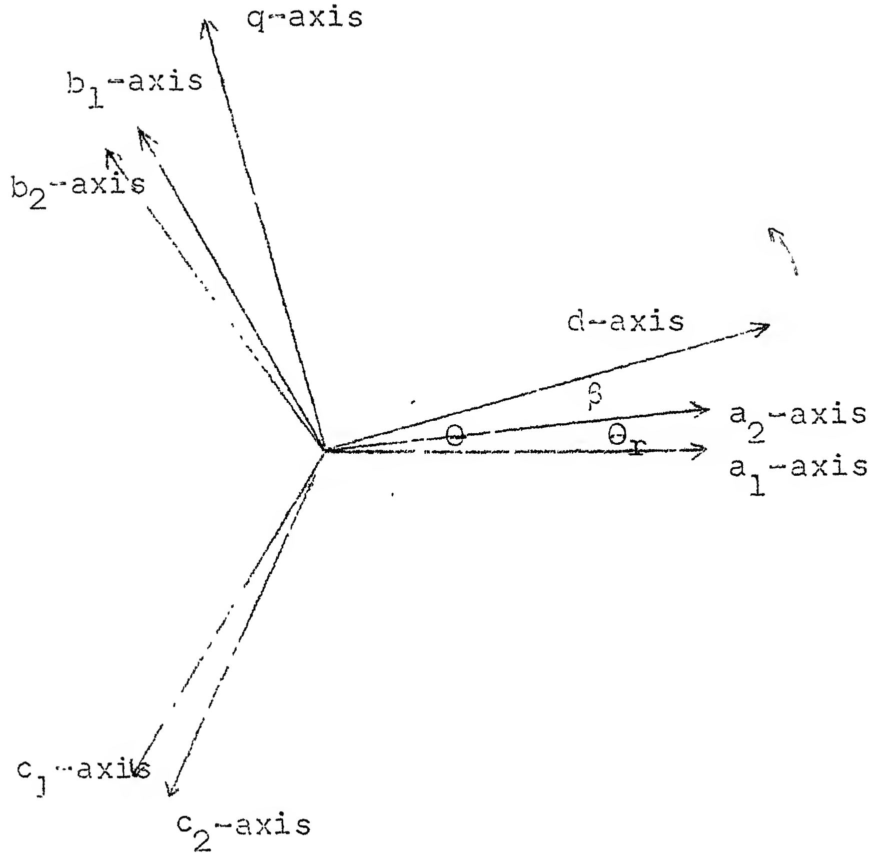


Fig. 1.1 Axes of three phase symmetrical machine

CHAPTER 2

STEADY STATE ANALYSIS OF THREE PHASE INDUCTION MOTOR FED BY AN IDEAL THREE PHASE CURRENT SOURCE INVERTER

2.1 INTRODUCTION

The three phase squirrel cage induction motor fed by an ideal square wave current has been analysed in this chapter. Fig. 2.1 shows the waveforms of the line current in the motor produced by ideal current source inverter. The dq model of the induction motor, with axis attached to the stator (Sec.1.1), is used for the analysis. In the analysis, the steady state solution has been obtained. This solution is for a constant inverter frequency and a constant rotor speed. The inverter frequency is adjustable by the inverter gating circuits.

The three phase current is transformed into the d-q current using the equations (1.9) and (1.10). The transformed d-q currents, for all the six intervals of the inverter period are listed in Table 2.1. In Table 2.1,

$$I = 2/3\ I_{dc} \tag{2.1}$$

where I_{dc} is the dc link current fed to the inverter.

The waveform of the d-q currents is given in the Fig. 2.1.

The waveform of the rotor current is obtained by the solution of equations (1.6). In the case of current fed induction motor, the above equations are decoupled from eqn. (1.7) which correspond to the phase voltage of the motor. This is in contrast with the voltage fed induction motor, in which case all the four equations of the matrix equations (1.6) and (1.7) have to be solved simultaneously.

Since we are solving for rotor currents at constant rotor speed, the problem can be viewed as obtaining the steady state solution of the differential equations with constant coefficients. This has been done using two well known techniques :

(1) Steady state solution in time domain determined by obtaining homogeneous and particular solutions and using two boundary value conditions.

(2) Steady state solution in frequency domain obtained by considering the input as the summation of sinusoidal signals, through fourier series of stator current and calculation of the transfer function for these inputs from the equation (1.6).

Equation (1.7) has been used in this chapter to obtain the voltage waveform in the d-q frame at the stator terminals. These equations have terms containing 'pi_{d1}' and 'pi_{q1}'. As 'i_{d1}' and 'i_{q1}' are discontinuous (Fig. 2.1) the voltage waveform is shown to have impulse.

Section 2.2 deals with the calculation of the rotor currents, using the time domain analysis. In this the cases of the stationary and the revolving rotors are dealt with separately. In case of the stationary rotor, the motor equations relating the dq stator and rotor currents, from equation (1.6) turn out to be first order and decoupled. The discontinuities in the rotor currents due to discontinuities in the stator current can be computed from equation (1.6).

In case of revolving rotor, equation (1.6) has two coupled equations for the rotor dq currents, i_{d2} and i_{q2}. A single equation for i_{d2} or i_{q2} can be obtained from equation (1.6) by elimination. However, the resulting single variable equation is a second order equation which contains the terms as '$p^2 i_{d1}$' and '$p^2 i_{q1}$'. Since the stator dq currents, i_{d1} and i_{q1} are discontinuous, these terms correspond to the derivatives of the impulses. To avoid such terms, the two new variables X_1 and X_2, have been defined instead of the rotor current variables, i_{d2} and i_{q2}. These variables X_1 and X_2 have been shown to be continuous and are proportional to the flux linkage in the machine. These are sometimes referred to as pseudo rotor currents in the literature [4].

The solution of the equation (1.6) in time domain requires two boundary conditions. These have been obtained by noting the fact that under steady state at each multiple of 60° in ωt, ω being the inverter frequency, is similar. The

stator mmf jumps in space by 60° due to the particular form of i_a, i_b and i_c (Fig. 2.1) which have discontinuities. The stator mmf instead of rotating smoothly as for sine wave currents jumps by 60° in space for every T/6 period of the inverter, T being the inverter time period. Thus, under steady state the rotor mmf must also move by 60° in space in each T/6 interval so that each interval is similar. The resulting boundary value relations based on above arguments are given in equations (2.5) and (2.6),

In Section 2.2, the solution for the initial value of rotor currents has been obtained independently for each interval. From these it can be seen the current solution obtained at the start of each interval is same as that obtained for the end of the previous interval. Thus it is sufficient to solve for the initial value of rotor current in any one of the six intervals.

Section 2.3 deals with the technique of frequency domain solution. For the rotor currents in this scheme the expressions for the rotor harmonic currents have been obtained in terms of the harmonic components of the stator current. The cases of stationary and rotating rotor have been dealt with in one general expression.

In Section 2.4, the voltage expressions in dq frame at the stator terminals have been obtained in time domain. The results of Section 2.2 and matrix equation (1.7) have been used for the

calculation. For the verification of the solutions through both the techniques for rotor currents, two computer program have been developed. The first program computes the time domain solution of the rotor current and then obtains the harmonic components of this solution. The second programs directly computes the harmonic components of the rotor current by the frequency domain technique (Section 2.3). In <u>Section 2.5</u> the case of a particular induction motor has been studied. Using the parameters of this induction motor, it has been shown that results obtained through both programs are in close agreement. This verifies these results.

2.2 ANALYSIS OF INDUCTION MOTOR FOR ROTOR CURRENTS THROUGH TIME DOMAIN

In this section, the steady state solutions through the time domain technique, Section 2.1, for the rotor currents have been obtained. For this the motor performance equation (1.6) has been solved. The cases of stationary and the rotating rotor have been dealt with separately. The equation (1.6) has been solved for the first three intervals (I,II,III) $o \leq \omega t \leq \pi$. The stator current waveform (Fig. 2.1) is symmetric about $\omega t = \pi$. Therefore, solutions for intervals, IV,V and VI, $\pi \leq \omega t \leq 2\pi$, are negative of the solutions of the intervals I,II and III respectively.

For the analysis, each interval has been associated with the discontinuity at the start of the interval. While solving equation (1.6), each interval has been divided into two sub-intervals. One corresponds to the point of discontinuity at th start of the interval and the other to the remaining period of that interval.

The following symbols have been used for the analysis.

$$I_{do} = i_{d2}(t_i = 0^-) \quad ; \quad I_{qo} = i_{q2}(t_i = 0^-)$$

$$I'_{d2} = i_{d2}(t_i = 0^+) \quad ; \quad I'_{q2} = i_{q2}(t_i = 0^+) \qquad (2.2)$$

$$I''_{d2} = i_{d2}(t_i = T_c) \quad ; \quad I''_{q2} = i_{q2}(t_i = T_c)$$

where T_c is the duration of each interval, i.e. ($T_c = \pi/3\omega$). The time 't_i' is the time within the interval where i denotes the interval number i.e. i = 1,2,3,. Thus t_1, t_2 and t_3 are defined for intervals I, II and III respectively and in these we have ($t_1 = t$), ($t_2 = (t-\pi/3\omega)$) and ($t_3 = (t-2\pi/3\omega)$). As already mentioned that each interval is associated with the discontinuity of stator current at the start of that interval. Thus ($t_i = 0^-$) refers to time just before the discontinuity has been considered in interval 'i' and ($t_i = 0^+$) corresponds to the time just after the discontinuity has been accounted for.

2.2.1 Case of Stationary Rotor

From equation (1.6), the equations relating the dq stator

and rotor currents for the stationary rotor ($\omega_r = 0$) are,

$$M p i_{d1} + (r_2 + L_{22} p) i_{d2} = 0 \tag{2.3}$$

$$M p i_{q1} + (r_2 + L_{22} p) i_{q2} = 0 \tag{2.4}$$

In this sub-section we are first obtaining the solution of i_{q2} and i_{d2} from above equations with the arbitary initial values of I_{qo} and I_{do}. Then, the fact that steady state dq currents at the end of each inverter period make the airgap mmf wave turn $\pi/3$ electrical radians, is used to compute I_{do} and I_{qo}.

The values of dq currents at the end of the inverter period are related to the initial values by the following equations [4], [11]

$$I''_{d2} = (I_{do} - \sqrt{3}\, I_{qo})/2 \tag{2.5}$$

$$I''_{q2} = (I_{qo} + \sqrt{3} I_{do})/2 \tag{2.6}$$

where I''_{d2} and I''_{q1} are as defined in equation (2.2).

In Section 2.2.1.1, the rotor currents expressions for interval I have been computed. Sections 2.2.1.2 and 2.2.1.3 give the rotor current expressions for intervals II and III respectively, which can be obtained by analysis as done in 2.2.1.1. Sec. 2.2.1.4 shows that it is sufficient to calculate I_{do} and I_{qo} for one interval only.

2.2.1.1 Interval I ($o \leq \omega t < \pi/3$)

As $t_1 = t$ in this interval, the time is denoted as t in this interval.

Calculation of currents at $t = 0$

From Table 2.1, at $t = 0$,

$$\Delta i_{d1}(t=o) = i_{d1}(t=0^+) - i_{d1}(t=0^-) = 0 \qquad (2.7)$$

$$\Delta i_{q1}(t=o) = i_{q1}(t=0^+) - i_{q1}(t=0^-) = -\sqrt{3}I \qquad (2.8)$$

Integrating equation (2.3) from $t = 0^-$ to $t = 0^+$,

$$M \int_{t=0^-}^{0^+} (pi_{d1})dt + r_2 \int_{t=0^-}^{0^+} i_{d2}dt + L_{22} \int_{t=0^-}^{0^+} (pi_{d2})dt = 0$$

Since i_{d2} is finite, this gives

$$M[i_{d1}(t=0^+) - i_{d1}(t=0^-)] + L_{22}[i_{d2}(t=0^+) - i_{d2}(t=0^-)] = 0$$

So from (2.7)

$$M\Delta i_{d1}(t=0) + L_{22}\Delta i_{d2}(t=0) = 0 \qquad (2.9)$$

So from equations (2.2), (2.7) and (2.9)

$$I'_{d2} = I_{do} \qquad (2.10)$$

Similarly from (2.4) using (2.4) and (2.3)

$$i_{q2}(t=0) = -\frac{M}{L_{22}} \times i_{q1} \tag{2.11}$$

So,

$$I'_{q2} = I_{qo} + \sqrt{3}\ K'_1 \tag{2.12}$$

where

$$K'_1 = \frac{M}{L_{22}} \tag{2.13}$$

Calculation of currents for $t > 0$

From Table 2.1, during this period

$$i_{d1} = -3I/2 \tag{2.14}$$

$$i_{q1} = -\sqrt{3}I/2 \tag{2.15}$$

Solving (2.3) and (2.4) using (2.14) and (2.15)

$$i_{d2}(t) = I'_{d2}\ \exp(-at) \tag{2.16}$$

$$i_{q2}(t) = I'_{q2}\ \exp(-at) \tag{2.17}$$

where

$$a = r_2/L_{22} \tag{2.18}$$

and I'_{q2} and I'_{d2} are as defined in equations (2.2).

Calculation of I_{do} and I_{qo}

Substituting $(t = \pi/3\omega)$ in equations (2.16) and (2.17) and using (2.10) and (2.12) we have

$$I''_{d2} = I_{do} \exp(-aT_c) \tag{2.19}$$

$$I''_{q2} = (I_{qo}+\sqrt{3}K_1) \exp(-aT_c) \tag{2.20}$$

Solving for I_{do} and I_{qo} from equations (2.5), (2.6), (2.19) and (2.20),

$$I_{do} = \frac{3}{2} \frac{k'_1 k'_2}{C_4} \tag{2.21}$$

$$I_{qo} = \frac{-\sqrt{3}\ k'_1\ k'_2}{C_4} [\exp(-aT_c) - 1/2) \tag{2.22}$$

where

$$k'_2 = \exp(-aT_c) \tag{2.23}$$

$$C_4 = \exp(-2aT_c)-\exp(-aT_c)+1 \tag{2.24}$$

2.2.1.2 Interval II (i.e. $\frac{\pi}{3} \leq \omega t < \frac{2\pi}{3}$ or $o \leq t_2 < \frac{\pi}{3}$)

The rotor current results have been obtained for this interval proceding similar to Sec. 2.2.1.1,

At $t_2 = 0$

$$I'_{d2} = I_{do} - \frac{3}{2} k'_1$$

$$I'_{q2} = I_{qo} + \frac{\sqrt{3}}{2} k'_1$$

For $t_2 > 0$

$$i_{d2}(t_2) = I'_{d2} \exp(-at_2) \tag{2.25}$$

$$i_{q2}(t_2) = I'_{q2} \exp(-at_2) \tag{2.26}$$

and

$$I_{do} = \frac{3}{2} \frac{k'_1 k'_2}{C_4} [\exp(-aT_c)] \tag{2.27}$$

$$I_{qo} = \frac{\sqrt{3} K'_1 K'_2}{C_4} [1 - \frac{\exp(-aT_c)}{2}] \tag{2.28}$$

2.1.3 Interval III (i.e. $\frac{2\pi}{3} \leq t < \pi$ or $o \leq t_3 < \frac{\pi}{3}$)

Similarly, for this interval,

at $t_3 = 0$

$$I'_{d2} = I_{do} - \frac{3}{2} K'_1$$

$$I'_{q2} = I_{qo} - \frac{\sqrt{3}}{2} K'_1 \tag{2.29}$$

for $t_3 > 0$

$$i_{d2}(t_3) = I'_{d2} \exp(-at_3) \quad (2.30)$$

$$i_{q2}(t_3) = I'_{q2} \exp(-at_3) \quad (2.31)$$

and

$$I_{do} = \frac{3}{2} \cdot \frac{K'_1 \cdot K'_2}{C_4} [\exp(-aT_c)-1)]$$

$$(2.32)$$

$$I_{qo} = \frac{\sqrt{3}}{2} \frac{K'_1 K'_2}{C_4} [\exp(-aT_c)+1]$$

2.2.1.4 Discussion

The rotor current expressions for intervals IV,V and VI can be observed directly from those in ~~Sections~~ intervals I,II and III respectively because of symmetry property. We also observe that for this case of stationary rotor, the closed form expressions for I_{do} and I_{qo} are obtained. In the above sections, the analytical expressions of I_{do} and I_{qo} have been computed independently for each interval. The expressions of the final values, at the end of each interval, can be obtained from equations (2.5) and (2.6) using the expressions of I_{do} and I_{qo} obtained for that interval. These values are summarised in Table 2.2.

It is seen from Tables 2.2 that the rotor currents at the start of each interval i.e. at ($t_i = 0$) is same as that

obtained at the end of previous interval i.e. at ($t_{(i-1)} = T_c$). Thus we need to solve for I_{do} and I_{qo} for one interval only.

2.2.2 Case of Rotating Rotor

From equation (1.6), the equations relating the dq stator and rotor currents with ω_r as the angular velocity of the rotor in electrical radians per second are

$$-M\omega_r\, i_{d1} + Mpi_{q1} + (r_2+L_{22}p)i_{q2} - L_{22}\omega_r i_{d2} = 0 \tag{2.33}$$

$$Mpi_{d1} + M\omega_r\, i_{q1} + L_{22}\omega_r i_{q2} + (r_2+L_{22}p)i_{d2} = 0 \tag{2.34}$$

These equations are coupled in contrast with the stationary rotor case, equations (2.3) and (2.4) where these are decoupled. Further pi_{d1} and pi_{q1} are impulse functions as i_{d1} and i_{q1} are discontinuous. The solution due to impulse functions has been done as follows :

To obtain the changes in rotor currents at any time instant 't_o' due to change in stator current we integrate equation (2.33) from $t = t_o^-$ to $t = t_o^+$. This gives

$$-M\int_{t=t_o^-}^{t_o^+} \omega_r\, i_{d1}dt + M\int_{t=t_o^-}^{t_o^+} pi_{q1}dt + r_2\int_{t=t_o^-}^{t_o^+} i_{q2}dt + L_{22}\int_{t=t_o^-}^{t_o^+} p\, i_{q2}dt$$

$$- L_{22}\int_{t=t_o^-}^{t_o^+} \omega_r\, i_{d2}\, dt = 0 \tag{2.37}$$

As currents are finite,

$$\int_{t=t_o^-}^{t_o^+} i_x \, dt = 0 \quad \text{where } x = q_2,\ d_1 \text{ or } d_2 \tag{2.38}$$

Thus, from (2.37), (2.38)

$$M[i_{q1}(t=t_o^+) - i_{q1}(t=t_o^-)] + i_{22}[i_{q2}(t=t_o^+) +$$

$$- i_{q2}(t=t_o^-)] = 0 \tag{2.39}$$

$$\Delta i_{q2}(t=t_o) = - \frac{M}{L_{22}} \cdot \Delta i_{q1}(t=t_o) \tag{2.40}$$

where $(t=t_o)$ corresponds to any arbitary time instant and 'Δ' corresponds to a jump in the current variable. Similarly, from equation (2.34)

$$\Delta i_{d2}(t=t_o) = - \frac{M}{L_{22}} \cdot \Delta i_{d1}(t=t_o) \tag{2.41}$$

Thus the effect of the discontinuity in i_{d1} and i_{q1} at $t_i = 0$ is a corresponding discontinuity in i_{d2} and i_{q2}, given by equations (2.41) and (2.40) respectively. This is equivalent to stating that the response due to 'pi_{d1}' and 'pi_{q1}' impulses in equations (2.33) and (2.34) can be looked upon as sudden changes in the initial conditions of i_{d2} and i_{q2}. Therefore, it is possible to solve (2.33) and (2.34) as a set of simultaneous equations. For the interval $t_i > o$, because i_{d1} and

i_{q1} are constants (Fig. 2.1) and pi_{d1}, pi_{q1} are zeros. Thus, a solution similar to Sec. 2.2 of stationary rotor can be obtained.

However, equations (2.33) and (2.34) can be solved more conveniently if a change of variable is done from i_{d2} and i_{q2} to X_2 and X_1 respectively. These variables, X_2 and X_1 are defined as,

$$X_1 = i_{q2} + \frac{M}{L_{22}} \; i_{q1} \tag{2.42}$$

$$X_2 = i_{d2} + \frac{M}{L_{22}} \; i_{d1} \tag{2.43}$$

It is shown in the following that these new variables X_1 and X_2 are continuous functions of time.

The change in X_1 at a arbitary point $(t = t_o)$ can be obtained from equction (2.42) as

$$\Delta X_1(t=t_o) = \Delta i_{q2}(t=t_o) + \frac{M}{L_{22}} \Delta i_{q1}(t=t_o) \tag{2.44}$$

Using (2.40) in (2.44) we get

$$\Delta X_1(t=t_o) = 0 \tag{2.45}$$

Similarly from (2.43) and (2.41)

$$\Delta X_2(t=t_o) = 0 \tag{2.46}$$

These new variables X_1 and X_2 are referred to as pseudo-rotor currents [4] and are proportional to the rotor flux in the machine.

In equations (2.33) and (2.34), eliminating ~~for~~ 'pi_{d2}' and 'pi_{q2}' by using (2.42) and (2.43), we obtain

$$pX_1 = \omega_r X_2 - ai_{q2} \tag{2.47}$$

$$pX_2 = -\omega_r X_1 - ai_{d2} \tag{2.48}$$

where

$$a = r_2/L_{22} \tag{2.49}$$

Substituting for i_{d2} and i_{q2} in equation (2.48) and (2.49) from (2.4~~4~~2) and (2.4~~5~~3) gives

$$pX_1 = \omega_r X_2 - aX_1 + \frac{M}{L_{22}}\, ai_{q1} \tag{2.50}$$

$$pX_2 = -\omega_r X_1 - aX_2 + \frac{M}{L_{22}}\, ai_{d1} \tag{2.51}$$

Differentiating (2.~~47~~50) and substituting for pX_2 from (2.4~~9~~7) and similarly differentiating (2.51) and substituting for pX_1 from equation (2.48), gives

$$(p^2+2ap+a^2+\omega_r^2)X_1 = K_1 ai_{q1}+K_1\omega_r i_{d1}+K_1 pi_{q1} \tag{2.52}$$

$$(p^2+2ap+a^2+\omega_r^2)X_2 = K_1 ai_{d1}-K_1\omega_r i_{q1}+K_1 pi_{d1} \tag{2.53}$$

where $K_1 = Ma/L_{22}$ (2.53a)

Equations (2.52) and (2.53) are the final equations which are solved to obtain rotor current expressions.

2.2.2.1 Interval - I ($o \leq \omega t < \pi/3$)

At $\omega t = 0$, from continuous property of X_1, X_2

$$\Delta X_1 = 0 \quad ; \quad \Delta X_2 = 0 \tag{2.54}$$

For $o < \omega t < \pi/3$, from Table 2.1

$$i_{d1} = -3I/2$$

$$i_{q1} = -\sqrt{3}I/2 \tag{2.55}$$

From (2.52), for this interval

$$(p^2+2ap+a^2+\omega_r^2)\ X_1 = -\frac{K_1 a\sqrt{3}I}{2} - \frac{K_1\omega_r 3I}{2}$$

The solution of above is

$$X_1(t) = \exp(-at)\ [C'_{o1}\ \sin\omega_r t + D'_{o1}\ \cos\omega_r t)] - K'_4 \tag{2.56}$$

where

$$K_4 = \frac{\sqrt{3}}{2}\ \frac{K_1 I(\sqrt{3}\omega_r+a)}{(a^2+\omega_r^2)} \tag{2.57}$$

and

C'_{o1}, D'_{o1} are constants.

If

$$X_{10} = X_1\Big|_{t_i=o} \quad ; \quad X_{20} = X_2\Big|_{t_i=o}$$

$$\dot{X}_{10} = \frac{dX_1}{dt}\Big|_{t_i=o} \quad ; \quad \dot{X}_{20} = \frac{dX_2}{dt}\Big|_{t_i=o} \tag{2.58}$$

$$X''_1 = X_1\Big|_{t_i=T_c} \quad ; \quad X''_2 = X_2\Big|_{t_i=T_c}$$

where i corresponds to interval. Here $i = 1$ and $t_1 = t$.

Using above in (2.56) gives

$$D'_{o1} = X_{10} + K_4 \tag{2.59}$$

$$C'_{o1} = (\dot{X}_{10}+aD'_{o1})/\omega_r \tag{2.60}$$

Solving (2.50) at $t = o$, using (2.55) and (2.58)

$$\dot{X}_{10} = \omega_r X_{20} - aX_{10} - \frac{\sqrt{3}}{2} K_1 I \tag{2.61}$$

Similarly solving (2.53) for X_2 gives

$$X_2(t) = \exp(-at)\ [C'_{o2} \sin \omega_r t + D'_{o2} \cos \omega_r t] + K_6 \tag{2.62}$$

where

$$K_6 = \frac{\sqrt{3}}{2} \frac{K_1 I(\omega_r - a\sqrt{3})}{(a^2+\omega_r^2)} \tag{2.63}$$

$$D'_{o2} = X_{20} - K_6 \tag{2.64}$$

$$C'_{o2} = [\dot{X}_{20}+aD'_{o2}]/\omega_r \tag{2.65}$$

$$\dot{X}_{20} = -\omega_r X_{10} - aX_{20} - \frac{3}{2} K_1 I \tag{2.66}$$

Computing X_1 at $t = T_c$ from (2.62), using (2.58)

$$X_1\big|_{t=T_c} = X_1'' = C'_{o1} Y_3 + D'_{o1} Y_4 - K_4 \tag{2.67}$$

where

$$Y_3 = \exp(-aT_c) \sin \omega_r T_c \tag{2.68}$$

$$Y_4 = \exp(-aT_c) \cos \omega_r T_c \tag{2.69}$$

Substituting for C'_{o1}, D'_{o1} in (2.67) from equations (2.59) and (2.60). Using (2.61), we have

$$X_1'' = Y_4X_{10} + Y_3X_{20} + Z_3' \tag{2.70}$$

where

$$Z_3' = C_5'Y_3 - K_4+K_4Y_4 \tag{2.71}$$

where

$$C_5' = [-\tfrac{\sqrt{3}}{2} K_1I + K_4a]/\omega_r \tag{2.72}$$

Similarly computing X_2 at $t = T_c$ and solving as above we get

$$X_2'' = -Y_3X_{10} + Y_4X_{20} + Z_4' \tag{2.73}$$

where

$$Z_4' = C_6'Y_3 - K_6Y_4 + K_6 \tag{2.74}$$

where

$$C_6' = [-\tfrac{3}{2} K_1I - aK_6]/\omega_r \tag{2.75}$$

Writing equations (2.70) and (2.73) in matrix form

$$\begin{bmatrix} X_2'' \\ X_1'' \end{bmatrix} = \begin{bmatrix} Y_4 & -Y_3 \\ Y_3 & Y_4 \end{bmatrix} \begin{bmatrix} X_{20} \\ X_{10} \end{bmatrix} + \begin{bmatrix} Z_4' \\ Z_3' \end{bmatrix} \tag{2.76}$$

From the boundary condition property, the relation between the initial and final values of current of an inverter interval,

$$\begin{bmatrix} X_2'' \\ X_1'' \end{bmatrix} = \begin{bmatrix} 1/2 & -\sqrt{3}/2 \\ \sqrt{3}/2 & 1/2 \end{bmatrix} \begin{bmatrix} X_{20} \\ X_{10} \end{bmatrix} \tag{2.77}$$

From equations (2.76) and (2.77), solving for X_{20}, X_{10}

$$\begin{bmatrix} X_{20} \\ X_{10} \end{bmatrix} = \begin{bmatrix} (\frac{1}{2} - Y_4) & -(\frac{\sqrt{3}}{2} - Y_3) \\ (\frac{\sqrt{3}}{2} - Y_3) & (\frac{1}{2} - Y_4) \end{bmatrix}^{-1} \begin{bmatrix} Z'_4 \\ Z'_3 \end{bmatrix} \qquad (2.78)$$

X_{20} and X_{10} are known from (2.78) after calculation of constan in the order K_r, K_4, K_6, Y_3, Y_4, C''_5, C'_6, Z'_3 and Z'_4.

The pseudo currents X_1 (t) and $X_2(t)$ are now known from equations (2.56) and (2.62). The actual rotor currents i_{d2} and i_{q2} can be obtained from equations (2.44) and (2.45).

2.2.2.2 Interval II (i.e. $\frac{\pi}{3} \leq \omega t < \frac{2\pi}{3}$ or $o \leq t_2 < \frac{\pi}{3}$)

Solving as interval I we obtain

$$X_1(t_2) = \exp(-at_2)[\cos \sin \omega_r t_2 + D'_{o3} \cos \omega_r t_2] - K_7 \qquad (2.79)$$

where

$$K_7 = \frac{\sqrt{3}\ K_1\ Ia}{(a^2+\omega_r^2)} \qquad (2.80)$$

$$D'_{o3} = X_{10} + K_7 \qquad (2.81)$$

$$C'_{o3} = \dot{X}_{10} + aD_{o3} \qquad (2.82)$$

and

$$X_2(t_2) = \exp(-at_2)[Co'_4 \sin \omega_r t_2 + D_{o4} \cos \omega_r t_2] + K_{10} \qquad (2.83)$$

where

$$K_{10} = \frac{\sqrt{3}I\ K_1\omega_r}{(a^2+\omega_r^2)} \qquad (2.84)$$

$$D'_{04} = X_{20} - K_{10} \tag{2.85}$$

$$C'_{04} = [\dot{X}_{20} + aD_{04}]/\omega_r \tag{2.86}$$

It should be noted that X_{10}, X_{10}, $\dot{X}_{10}$ and $\dot{X}_{20}$ correspond to definition of equation (2.58) with i = 2.

For initial condition of this interval we have the relation

$$\begin{bmatrix} X_{20} \\ X_{10} \end{bmatrix} = \begin{bmatrix} (\frac{1}{2} - Y_4) & -(\frac{\sqrt{3}}{2} - Y_3) \\ (\frac{\sqrt{3}}{2} - Y_3) & (\frac{1}{2} - Y_4) \end{bmatrix}^{-1} \begin{bmatrix} Z'_8 \\ Z'_7 \end{bmatrix} \tag{2.87}$$

where

$$Z'_8 = C'_{12}\, Y_3 - K_{10}Y_4 + K_{10} \tag{2.88}$$

$$Z'_7 = C'_{11}\, Y_3 + K_7\, Y_4 - K_7 \tag{2.89}$$

$$C'_{12} = -K_{10}\, a/\omega_r \tag{2.90}$$

$$C'_{11} = [-\sqrt{3}\, K_1 I + K_7 a]/\omega_r \tag{2.91}$$

Once X_{10} and X_{20} are solved, the pseudo currents $X_1(t)$ and $X_2(t)$ are defining over the entire interval $\pi/3$ to $2\pi/3$. The actual rotor currents can be found out using equations (2.44) and (2.45).

2.2.2.3 Interval III (i.e. $\frac{2\pi}{3} \leq \omega t < \pi$ or $0 \leq t_3 < \frac{\pi}{3}$)

Solving as for interval I we obtain for this interval.

$$X_1(t) = \exp(-at_3)[\cos' \sin \omega_r t_3 + D'_{o5} \cos \omega_r t_3] + K_{13} \quad (2.92)$$

$$X_2(t) = \exp(-at_3)[Co'_6 \sin \omega_r t_3 + D'_{o6} \cos \omega_r t_3] + K_{16} \quad (2.93)$$

where

$$t_3 = t - 2\pi/3 \quad (2.94)$$

$$K_{13} = \frac{\sqrt{3}K_1 I}{2} \frac{(\sqrt{3}\omega_r - a)}{(a^2+\omega_r^2)} \quad (2.95)$$

$$K_{16} = \frac{\sqrt{3}K_1 I}{2} \frac{(\omega_r+\sqrt{3}a)}{(a^2+\omega_r^2)} \quad (2.96)$$

$$D'_{o5} = X_{10} - K_{13} \quad (2.97)$$

$$D'_{o5} = [\dot{X}_{10} + aD_{o5}]/\omega_r \quad (2.98)$$

$$D'_{o6} = X_{20} - K_{16} \quad (2.99)$$

$$C'_{o6} = [\dot{X}_{20} + aD_{o6}]/\omega_r \quad (2.100)$$

For the initial condition, the following relation is obtained

$$\begin{bmatrix} X_{20} \\ X_{10} \end{bmatrix} = \begin{bmatrix} (\frac{1}{2} - Y_4) & -(\frac{\sqrt{3}}{2} - Y_3) \\ (\frac{\sqrt{3}}{2} - Y_3) & (\frac{1}{2} - Y_4) \end{bmatrix}^{-1} \begin{bmatrix} Z'_{12} \\ Z'_{11} \end{bmatrix} \quad (2.101)$$

where

$$Z'_{12} = C'_{18}\, Y_3 - K_{16}\, Y_4 + K_{16} \qquad (2.102)$$

$$Z'_{11} = C'_{17}\, Y_3 - K_{13}\, Y_4 + K_{13} \qquad (2.103)$$

where

$$C'_{17} = [- \frac{\sqrt{3}K_1 I}{2} - K_{13} a]/\omega_r \qquad (2.104)$$

$$C'_{18} = [\ \frac{3K_1 I}{2} - K_{16} a]/\omega_r \qquad (2.105)$$

Thus, the actual rotor currents have been found out for this interval. In rotating rotor case it can be seen that analytical expressions are complicated and closed form expressions are not obtained for dq rotor currents. A computer program has been developed to calculate initial and final values of currents for these intervals in rotating rotor case. The boundary conditions have been shown to be matching for the case of induction motor in Section 2.5.

2.3 ANALYSIS OF INDUCTION MOTOR FOR ROTOR CURRENTS THROUGH FREQUENCY DOMAIN

The expressions for different harmonics of the rotor currents have been computed in this section. The cases of stationary and rotating rotors have been dealt with in one general expression.

The analysis proceeds in two steps. Firstly, is the calculation of the harmonic components of the stator current. Then equations (2.52) and (2.53) are used to compute the harmonics of pseudo rotor currents X_1 and X_2. With this result, using equations (2.42) and (2.43), the harmonics of rotor currents can be obtained.

For calculation of the harmonic components of stator current, i_{d1} and i_{q1} (Fig. 2.2) are resolved into Fourier series as

$$i_{d1} = \sum_{n=1}^{\infty} a_{n_d} \cos n\omega t \tag{2.106}$$

$$i_{q1} = \sum_{n=1}^{\infty} b_{n_q} \sin n\omega t \tag{2.107}$$

where

for all even values of n

$$a_{n_d} = 0 \tag{2.108}$$

$$b_{n_q} = 0 \tag{2.109}$$

and for all odd values of n

$$a_{n_d} = - \frac{6I}{n\pi} \sin \frac{n\pi}{3} \tag{2.110}$$

$$b_{n_q} = - \frac{4\sqrt{3}I}{n\pi} \sin^2 \frac{n\pi}{3} \tag{2.111}$$

It can be seen from (2.110) and (2.111) that for all triplensi.e. n equal to a multiple of 3, a_{n_d} and $b_{n_q} = 0$. Thus

$$\text{for } n = 1,5,7,11,13,17, \ldots \tag{2.112}$$

$$a_{n_d} = -\frac{6I}{n\pi} \sin \frac{n\pi}{3} \tag{2.113}$$

$$b_{n_q} = -\frac{3\sqrt{3}}{n\pi} I \tag{2.114}$$

and all other harmonics are absent.

In the remainder of this section only odd and non triplent harmonics are considered from (2.106) and (2.107)

$$i_{d_1} = \sum_n -\frac{6I}{n\pi} \sin \frac{n\pi}{3} \cos n\omega t \tag{2.115}$$

$$i_{q_1} = \sum_n -\frac{3\sqrt{3}I}{n\pi} \sin n\omega t \tag{2.116}$$

Writing for mth harmonic of i_{d_1} and i_{q_1}

$$i_{d1_m} = I_{md} \cos(m\omega t + \alpha_{md}) \tag{2.117}$$

$$i_{q1_m} = I_{mq} \cos(m\omega t + \alpha_{mq}) \tag{2.118}$$

So, from (2.115) and (2.116)

$$|I_{md}| = |I_{mq}| = |I_m| \tag{2.119}$$

$$\alpha_{md} = 0 \tag{2.120}$$

$$\alpha_{mq} = -\pi/2 \text{ for } m = 1,7,13,19, \ldots \tag{2.121}$$

$$\alpha_{mq} = +\pi/2 \text{ for } m = 5,11,17,23,\ldots \tag{2.122}$$

We define a variable 'p' as

$$p = \begin{cases} 1 & \text{if } m = 1,7,13,19, \ldots \\ 0 & \text{if } m = 5,11,17,23,\ldots \end{cases} \tag{2.123}$$

then from (2.117) and (2.118)

$$i_{d1m} = I_m \cos m\omega t \tag{2.124}$$

$$i_{q1m} = I_m \cos (m\omega t + (-1)^p \pi/2) \tag{2.125}$$

where

$$I_m = (-1)^p \frac{3\sqrt{3}I}{m\pi} = a_{nd} \tag{2.126}$$

Solving (2.52) for mth harmonic of X_1 denoted by X_{1m}, equation becomes

$$(p^2+2ap+a^2+\omega_r^2)X_{1m} = K_1 a i_{q1m} + K_1 \omega_r i_{d1m} + K_1 p i_{q1m}$$

This equation can be written in phasor form as

$$(A_T\angle\alpha_T)\vec{X}_{pm} = K_1 a\vec{I}_{q1m}+K_1\omega_r\vec{I}_{d1m}+K_1(m\omega\angle 90^o)\ \vec{I}_{q1m} \qquad (2.127)$$

where

$$A_T = [(-m^2\omega^2+a^2+\omega_r^2)^2+(2a\omega m)^2]^{1/2} \qquad (2.128)$$

$$\alpha_T = \tan^{-1}\ [2am\omega/(a^2+\omega_r^2-m^2\omega^2)] \qquad (2.129)$$

$$\vec{I}_{q1m} = \frac{I_m}{\sqrt{2}}\ \angle(-1)^p\ \frac{\pi}{2} \qquad (2.130)$$

$$\vec{I}_{d1m} = \frac{I_m}{\sqrt{2}}\ \angle 0^o \qquad (2.131)$$

and $\vec{X}_{1m}$ corresponds to the phasor of $X_{1m}(t)$ from (2.127), (2.130) and (2.131)

$$\vec{X}_{1m} = (\frac{K_1 a}{A_T}\ \angle -\alpha_T)\ \vec{I}_{q1m} + (\frac{K_1\omega_r}{A_T}\ \angle -\alpha_T)\ \vec{I}_{d1m} + $$

$$+ (\frac{m\omega K_1}{A_T}\ \angle 90^o - \alpha_T)\ \vec{I}_{q1m} \qquad (2.132)$$

This can be visualised as if $\vec{X}_{1m}$ is obtained from summation of three phasors which can be obtained from the $\vec{I}_{q1m}$ and $\vec{I}_{d1m}$ by passing these through appropriate amplifier, as shown in Fig. 2.2.

To obtain $X_{1m}(t)$, taking the real part of equation (2.132), we obtain

$$X_{1m}(t) = \frac{K_1 a I_m}{A_T} \cos(m\omega t - \alpha_T + (-1)^p \pi/2) + \frac{K_1 \omega_r I_m}{A_T} \cos(m\omega t - \alpha_T) +$$

$$+ \frac{m\omega K_1}{A_T} I_m \cos(m\omega t + \frac{\pi}{2} + (-1)^P \frac{\pi}{2} - \alpha_T)$$

This gives

$$X_{1m} = -\frac{K_1 a(-1)^P}{A_T} I_m \sin(m\omega t - \alpha_T) + \frac{K_1 I_m}{A_T} (\omega_r - (-1)^P m\omega) \cos(m\omega t - \alpha_T) \tag{2.133}$$

Similarly we can obtain mth harmonic of X_2, defined as X_{2m} from equation (2.53) as

$$X_{2m} = \frac{K_1 a(-1)^P}{A_T} I_m \cos(m\omega t - \alpha_T) - \frac{K_1 I_m}{A_T} (\omega_r - (-1)^p \omega_r) \sin(m\omega t - \alpha_T) \tag{2.134}$$

It is noted from equations (2.123) and (2.134) that magnitudes of X_{1m} and X_{2m} are equal and they have a phase difference of $\pi/2$.

Thus the harmonic components of the pseudo rotor currents X_1 and X_2 are computed, Using equations (2.44) and (2.45) the harmonic components of the rotor current can be obtained.

Using the above procedure, the harmonics of the pseudo-rotor currents have been computed for a case of particular motor, in Sec. 2.5.

2.4 VOLTAGE WAVEFORM AT THE STATOR TERMINALS

In this section, the voltage waveform, in the dq frame, at the stator terminals has been computed from the equation (1.7),

$$v_{d1} = (r_1+L_{11}p)\ i_{d1} + M_p i_{d2} \tag{2.135}$$

$$v_{q1} = (r_1+L_{11}p)\ i_{q1} + M_p i_{q2} \tag{2.136}$$

The expressions for i_{q2} and i_{d2} obtained in Sec. 2.2 have been used in the above equations to compute the voltage waveform. The cases of stationary and rotating rotor have been dealt with separately.

2.4.1 For the Stationary Rotor

For the stationary rotor, substituting for pi_{d2} from equation (2.3) into equation (2.115) we have

$$V_{d1} = (r_1+L_{11}p)i_{d1}+M[-(\frac{M}{L_{22}}\ pi_{d1} + \frac{r_2}{L_{22}}\ i_{d2})]$$

Using $(a = r_2/L_{22})$ from equation (2.18), in above,

$$V_{d1} = r_1 i_{d1} - Ma\ i_{d1} + (L_{11} - \frac{M^2}{L_{22}})\ pi_{d1} \tag{2.137}$$

Similarly, from (2.4) and (2.136)

$$V_{q1} = r_1 i_{q1} - Mai_{q2} + (L_{11} - \frac{M^2}{L_{22}})pi_{q1} \tag{2.138}$$

It can be noted, that for ideal square wave current, pi_{d1} and pi_{q1} are zero, except at the end points corresponding to $\omega t = o, \pi/3, 2\pi/3, \ldots$.

At these point, V_{d1} and V_{q1} has impulse. From equation (2.117), it is observed that at these points of discontinuity of i_{d1}, V_{d1} has a impulse of the magnitude

$$\delta\,(t_i=o)\Big|_{V_{d1}} = A_1 \quad i_{d1}(t_i=o) \tag{2.139}$$

where

$$A_1 = (L_{11} - \frac{M^2}{L_{22}}) \tag{2.140}$$

Thus it is seen that there is an impulse in the voltage waveform because of the leakage inductance.

It is also observed from equation (2.137) that there a jump in V_{d1} also apart from impulse because of discontinuity of i_{d1} and i_{d2}. If by 'Δ' we refer to the change at the point of consideration, from (2.137) we have

$$\Delta V_{d1}(t_i=o) = r_1 \Delta i_{d1}(t_i=o) - Ma \Delta i_{d2}(t_i = o) \tag{2.141}$$

Substituting for Δi_{d2} $(t_i = o)$ from (2.9) into (2.141) we have

$$\Delta V_{d1}(t_i=o) = A_2 \Delta i_{d1}(t_i=o) \tag{2.142}$$

where

$$A_2 = (r_1 + \frac{M^2}{L_{22}} a) \tag{2.143}$$

Because ($a \alpha r_2$) and hence this jump is because of rotor and stator resistances which should be so as there is a sudden change of current through these resistances.

Similarly, for V_{q1} we have from (2.138) and (2.11)

$$\delta(t_i=o)\Big|_{V_{q1}} = A_1 \, \Delta i_{q1}(t_i=o) \tag{2.144}$$

$$\Delta V_{qi}(t_i=o) = A_2 \, \Delta i_{q1}(t_i=o) \tag{2.145}$$

During the time $(t_i > o)$ we note that i_{q1} and i_{d1} are constants and hence equations (2.137) and (2.138) reduce to

$$V_{d1} = r_1 i_{d1} - Mai_{d2} \quad \text{for} \quad t_i > o \tag{2.146}$$

$$V_{q1} = r_1 i_{q1} - Mai_{q2} \quad \text{for} \quad t_i > o \tag{2.147}$$

Thus, knowing stator and rotor currents expressions, the voltages V_{d1} and v_{q1} can be computed the impulse and jump magnitudes can be computed from the information of discontinuities of i_{q1} and i_{d1}. Table 2.3 gives the reference equation numbers which are are to be used to obtain voltage expression by use of equations (2.142), (2.144), (2.146) and (2.147). The table is only for interval I,II and III because expressions for intervals IV,V and VI can be obtained from these because of symmetry about $(\omega t = \pi)$.

2.4.2 For the Rotating Rotor

From equation (2.33), we have

$$pi_{q2} = \omega_r i_{d2} - ai_{q2} + \frac{M}{L_{22}} \omega_r i_{d1} - \frac{M}{L_{22}} pi_{q1}$$

Substituting for 'pi_{q2}' in (2.136) from this equation,

$$V_{q1} = r_1 i_{q1} - Ma\,i_{q2} + M\omega_r (M/L_{22}\; i_{d1} + i_{d2}) + A_1 p i_{q1} \tag{2.148}$$

where 'A_1' is given by the equation (2.140).

Similarly, from (2.34) and (2.135) we have

$$V_{d1} = r_1 i_{d1} - Ma\,i_{d2} - M\omega_r \left(\frac{M}{L_{22}}\; i_{q1} + i_{q2}\right) + A_1 p i_{d1} \tag{2.149}$$

From (2.148) it can be said that at the end points of the intervals (ωt = o, $\pi/3$, $2\pi/3$). V_{q1} has an impulse and a jump, whose expressions are given as

$$\delta\,(t_i = o)\Big|_{V_{q1}} = A_1 \Delta\, i_{q1}(t_i = o) \tag{2.150}$$

$$\Delta V_{q1}(t_i = o) = r_1 \Delta\, i_{q1} - Ma\,\Delta i_{q2} + M\omega_r \left(\frac{M}{L_{22}}\; \Delta\, i_{q1} + \Delta\, i_{q2}\right) \tag{2.151}$$

Substituting for $\Delta\, i_{q2}$ in terms of $\Delta\, i_{q1}$ and for Δi_{d2} in terms of $\Delta\, i_{d1}$ from equations (2.9) and (2.11) respectively, in (2.151) we have

$$\Delta V_{q1}(t_i = o) = A_2\; \Delta\, i_{q1}(t_i = o) \tag{2.152}$$

where A_2 is given by equation (2.143).

Similarly from (2.149) using (2.9) and (2.11)

$$\delta\,(t_i = o)\Big|_{V_{d1}} = A_1\; \Delta\, i_{d1}(t_i = o) \tag{2.153}$$

$$\Delta V_{d1}(t_i = o) = A_2\; \Delta\, i_{d1}(t_i = o) \tag{2.154}$$

Thus, the behaviour of v_{d1} and v_{q1} at the points of discontinuity of i_{q1} or i_{d1} can be computed from the equations (2.150), (2.152), (2.153) and (2.154).

For the remaining period of time, we note that i_{d1} and i_{q1} are constants, Fig. 2.2. So, <u>for $t_i > o$</u>, equation (2.148), (2.149) reduce to

$$V_{q1} = r_1 i_{q1} - Mai_{q2} + M\omega_r \left(\frac{M}{L_{22}} i_{d1} + i_{d2}\right) \tag{2.155}$$

$$V_{d1} = r_1 i_{d1} - Mai_{d2} - M\omega_r \left(\frac{M}{L_{22}} i_{q1} + i_{q2}\right) \tag{2.156}$$

From equations (2.42) and (2.43)

$$i_{q2} = X_1 - \frac{M}{L_{22}} i_{q1} \tag{2.157}$$

$$i_{d2} = X_2 - \frac{M}{L_{22}} i_{d1} \tag{2.158}$$

Substituting for i_{q2} and i_{d2} from equations (2.157) and (2.158) in equations (2.155) and (2.156) we have

$$V_{d1} = A_2\, i_{d1} - Ma\, X_2 - M\omega_r\, X_1 \tag{2.159}$$

$$V_{q1} = A_2\, i_{q1} - Ma\, X_1 + M\omega_r\, X_2 \tag{2.160}$$

Since X_1 and X_2 expressions have been computed in Sec. 2.2.2, the expressions for V_{d1} and V_{q1} can be computed.

Table 2.4 lists the reference to equations numbers of X_1 and X_2 during intervals I, II and III which are to be used in equations (2.159) and (2.160) to compute V_{dl} and V_{ql}. The table also lists the reference to changes in currents at $t_i = o$, which can be used in equations (2.150), (2.152), (2.153) and (2.154) to compute the magnitudes of impulses and jump at ($t_i = o$) for the waveform of V_{dl} and V_{ql}.

2.5 CALCULATIONS FOR A KNOWN MACHINE PARAMETERS

For the calculation, a slip ring induction motor considered in [12] has been taken. The parameters of the machine are given in Appendix A.

For this machine two computer programs are developed in DEC System 10 computer in FORTRAN. One uses the results of Section 2.2 to compute the sampled rotor current waveform in time domain. The other program computes the various harmonic components of the pseudo rotor current using Section 2.3 approach. The listings of these programs are given in Appendix B. These programs have inverter input dc current, inverter frequency and rotor speed as parameters.

Table 2.5 gives the values of currents at the start and end of intervals I, II and III using results of Sec.2.2.2 for rotating rotor case. It is seen from this table that the rotor currents at the start of each interval i.e. at (t_i=0)

is same as that obtained at the end of the previous interval i.e. at ($t_{(i-1)} = T_c$). Thus I_{do} and I_{qo} are to be computed only for one interval.

Table 2.6 gives the harmonic components of the time solution of pseudo rotor current X_1 obtained through first program. The harmonic components which are obtained by second program using Sec. 2.3 have also been listed. Both these components are seen to similar and this verifies that both the programs give same solution.

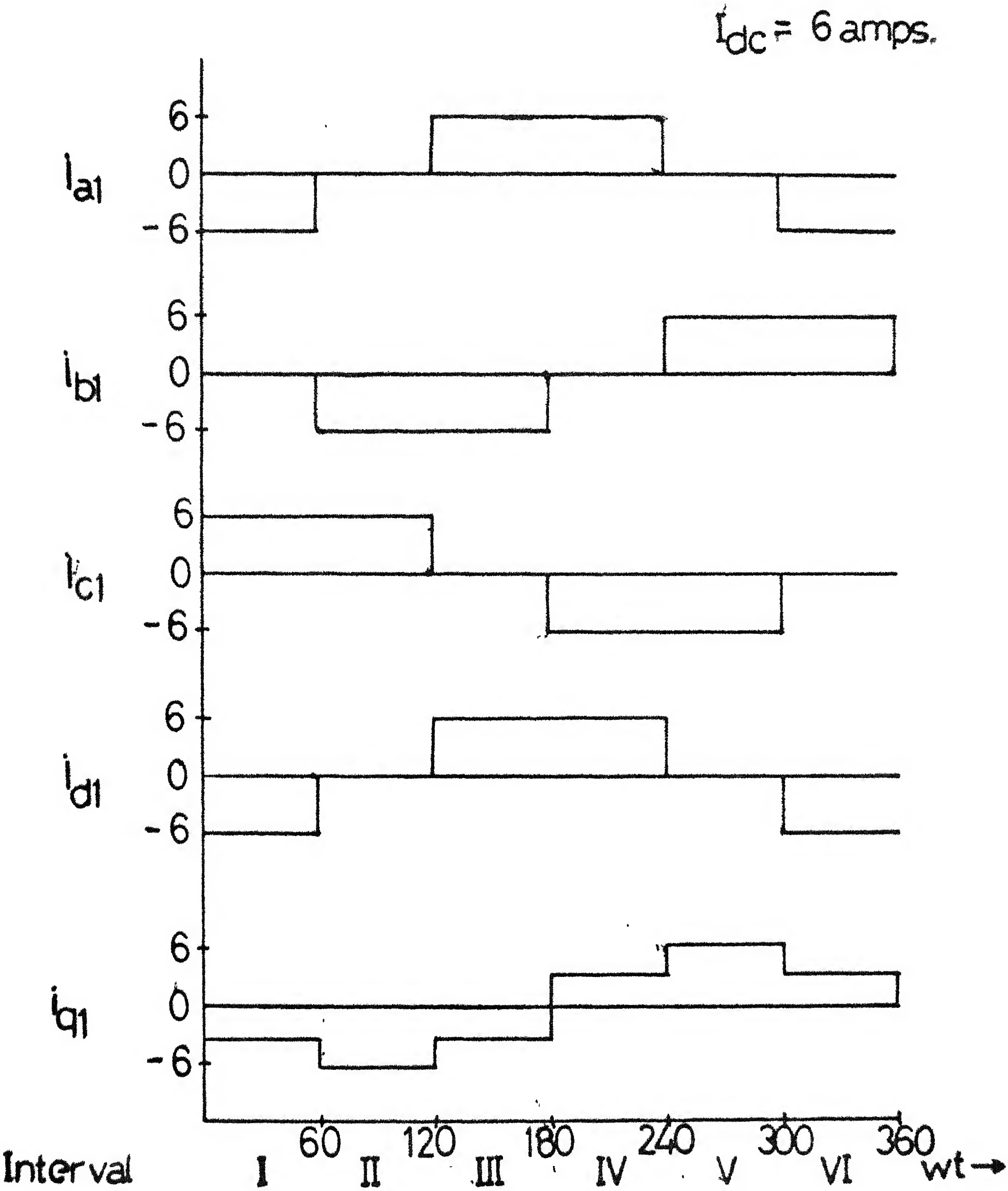


Fig 2.1: Three Phase & D Q Stator Currents

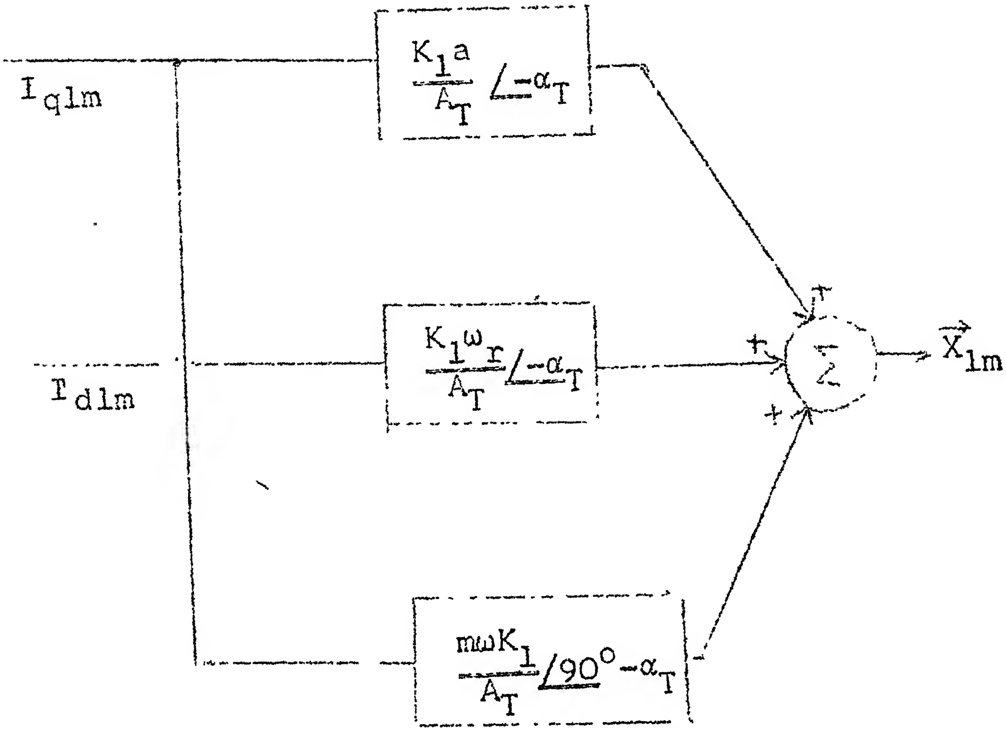


Fig. 2.2 Derivation of mth harmonic of X_1 from I_{d1} and I_{q1}

Table 2.1

Three phase and two phase stator currents

Interval/ conducting	3 phase currents	D-Q currents
I $0 \leq \omega t < \pi/3$	$i_{a1} = -I_{dc}$ $i_{b1} = 0$ $i_{c1} = I_{dc}$	$i_{d1} = -3I/2$ $i_{q1} = -\sqrt{3}I/2$ $\Delta i_{d1}(\omega t=0) = 0$ $\Delta i_{q1(\omega t=0)} = -\sqrt{3}I$
II $\frac{\pi}{3} \leq \omega t < \frac{2\pi}{3}$	$i_{a1} = 0$ $i_{b1} = -I_{dc}$ $i_{c1} = I_{dc}$	$i_{d1} = 0$ $i_{q1} = -\sqrt{3}I$ $\Delta i_{d1(\omega t=\pi/3)} = 3I/2$ $\Delta i_{q1(\omega t=\pi/3)} = -\sqrt{3}I/2$
III $\frac{2\pi}{3} \leq \omega t < \pi$	$i_{a1} = I_{dc}$ $i_{b1} = -I_{dc}$ $i_{c1} = 0$	$i_{d1} = 3I/2$ $i_{q1} = -\sqrt{3}I/2$ $\Delta i_{d1(\omega t=2\pi/3)} = 3I/2$ $\Delta i_{q1(\omega t=2\pi/3)} = \sqrt{3}I/2$
IV $\pi \leq \omega t < \frac{4\pi}{3}$	$i_{a1} = I_{dc}$ $i_{b1} = 0$ $i_{c1} = -I_{dc}$	$i_{d1} = 3I/2$ $i_{q1} = \sqrt{3}I/2$ $\Delta i_{d1(\omega t=\pi)} = 0$ $\Delta i_{q1(\omega t=\pi)} = \sqrt{3}I$
V $\frac{4\pi}{3} \leq \omega t < \frac{5\pi}{3}$	$i_{a1} = 0$ $i_{b1} = I_{dc}$ $i_{c1} = -I_{dc}$	$i_{d1} = 0$ $i_{q1} = \sqrt{3}I$ $\Delta i_{d1(\omega t=4\pi/3)} = -3I/2$ $\Delta i_{q1(\omega t=4\pi/3)} = \sqrt{3}I/2$
VI $\frac{5\pi}{3} \leq \omega t < 2\pi$	$i_{a1} = -I_{dc}$ $i_{b1} = I_{dc}$ $i_{c1} = 0$	$i_{d1} = -3I/2$ $i_{q1} = \sqrt{3}I/2$ $\Delta i_{d1(\omega t=5\pi/3)} = -3I/2$ $\Delta i_{q1(\omega t=5\pi/3)} = -\sqrt{3}I/2$

Table 2.2

Initial and final values of D-Q rotor currents for stationary motor

Interval	I_{do}	I_{qo}	I''_{q2}	I''_{q2}
$o \leq \omega t \leq \pi/3$ INTERVAL I	$\frac{3K'_2K'_1}{2C_4}$	$-\frac{\sqrt{3}K'_2K'_1}{C_4}$ $[\exp(-aT_c)-1/2]$	$\frac{3K'_2K'_1}{2C_4}\exp(-aT_c)$	$\frac{\sqrt{3}K'_2K'_1}{C_4}$ $[1-\exp(-aT_c)$
$\pi/3 \leq \omega t \leq 2\pi/3$ INTERVAL II	$\frac{3K'_2K'_1}{2C_4}\exp(-aT_c)$	$\frac{\sqrt{3}K'_2K'_1}{C_4}$ $[1-\exp(-aT_c)]$	$\frac{3K'_2K'_1}{2C_4}[\exp(-aT_c)-1]$	$\frac{\sqrt{3}K'_2K'_1}{2C_4}$ $[\exp(-aT_c)+1$
$2\pi/3 \leq \omega t \leq \pi$ INTERVAL III	$\frac{3K'_2K'_1}{2C_4}[\exp(-aT_c)-1]$	$\frac{\sqrt{3}K'_2K'_1}{2C_4}$ $[\exp(-aT_c)-1]$	$-\frac{3K'_2K'_1}{2C_4}$	$\frac{\sqrt{3}K'_2K'_1}{C_4}$ $[\exp(-aT_c)-1$

Table 2.3

Reference for the computation of stator voltage expressions in the case of stationary rotor

INTERVAL	For i_{d1} and i_{q1}	For i_{d2}	For i_{q2}	For Δi_{d1} and Δi_{q1} at $t_i = 0$
I	Table 2.1	eqn.(2.16)	eqn.(2.17)	Table 2.1
II	Table 2.1	eqn.(2.25)	eqn.(2.26)	Table 2.1
III	Table 2.1	eqn.(2.30)	eqn.(2.31)	Table 2.1

Table 2.4

Reference for the computation of voltage expressions of rotating rotor case

Interval	for i_{d1} and i_{q1}	for X_1	for X_2	for Δi_{d1} and Δi_{q1} at $t_i = 0$
I	Table 2.1	eqn.(2.56)	eqn. (2.62)	Table 2.1
II	Table 2.1	eqn.(2.79)	eqn. (2.83)	Table 2.1
III	Table 2.1	eqn.(2.92)	eqn. (2.93)	Table 2.1

TABLE 2.5
****

INTERVAL NUMBER	**FOR X1** INITIAL VALUE	FINAL VALUE	**FOR X2** INITIAL VALUE	FINAL VALUE
FOR IDC=6.AMP.;INV. FREQ.=20.HZ & RPM=500.				
I	2.96724	-0.12871	-1.87004	-3.50436
II	-0.13596	-3.09925	-3.50471	-1.64064
III	-3.10320	-2.96724	-1.63467	1.87002
FOR IDC=6.AMP.;INV. FREQ.=20.HZ & RPM=300.				
I	1.37357	0.44596	-0.28078	-1.32837
II	0.44359	-0.92744	-1.32993	-1.05037
III	-0.92998	-1.37355	-1.04915	0.28079
FOR IDC=8.AMP.;INV. FREQ.=30.HZ & RPM=500.				
I	1.39203	0.51240	-0.21476	-1.31125
II	0.51000	-0.87940	-1.31291	-1.09935
III	-0.88203	-1.39201	-1.09815	0.21476
FOR IDC=4.AMP.;INV. FREQ.=15.HZ & RPM=300.				
I	1.59855	0.22948	-0.66173	-1.71421
II	0.22617	-1.36982	-1.71524	-1.05581
III	-1.37239	-1.59854	-1.05352	0.66172

TABLE 2.6
****

HAR.	VIA TIME DOMAIN MAGNITUDE	VIA TIME DOMAIN PHASE	VIA FREQ. DOMAIN MAGNITUDE	VIA FREQ. DOMAIN PHASE
RPM = 100.000 IDC = 6.00 FREQ OF INVERTER = 20.0 HZ.				
1	0.81484	97.3408	0.81483	97.3425
5	0.02649	91.1843	0.02650	91.1907
7	0.01431	90.8909	0.01431	90.9004
11	0.00557	90.5369	0.00557	90.5510
13	0.00410	90.4622	0.00410	90.4795
17	0.00234	90.3363	0.00235	90.3585
19	0.00191	90.3017	0.00191	90.3267
RPM = 300.000 IDC = 6.00 FREQ OF INVERTER = 20.0 HZ.				
1	1.33879	102.1182	1.33876	102.1208
5	0.02488	91.1141	0.02489	91.1185
7	0.01505	90.9363	0.01504	90.9465
11	0.00541	90.5234	0.00541	90.5351
13	0.00421	90.4746	0.00421	90.4923
17	0.00230	90.3322	0.00230	90.3516
19	0.00195	90.3078	0.00195	90.3326
RPM = 500.000 IDC = 6.00 FREQ OF INVERTER = 20.0 HZ.				
1	3.45353	122.7886	3.45320	122.7928
5	0.02344	91.0557	0.02347	91.0547
7	0.01587	90.9864	0.01586	90.9977
11	0.00524	90.5151	0.00526	90.5200
13	0.00434	90.4881	0.00433	90.5057
17	0.00224	90.3338	0.00226	90.3451
19	0.00199	90.3163	0.00198	90.3387
RPM = 500.000 IDC = 6.00 FREQ OF INVERTER = 30.0 HZ.				
1	1.01392	99.1472	1.01389	99.1501
5	0.01642	90.7347	0.01643	90.7383
7	0.01012	90.6261	0.01012	90.6365
11	0.00359	90.3444	0.00359	90.3550
13	0.00282	90.3122	0.00282	90.3297
17	0.00153	90.2160	0.00153	90.2337
19	0.00130	90.1977	0.00130	90.2225
RPM = 500.000 IDC = 8.00 FREQ OF INVERTER = 30.0 HZ.				
1	1.35189	99.1472	1.35186	99.1501
5	0.02189	90.7347	0.02191	90.7383
7	0.01349	90.6260	0.01349	90.6365
11	0.00478	90.3445	0.00479	90.3550
13	0.00376	90.3122	0.00376	90.3297
17	0.00203	90.2159	0.00204	90.2337
19	0.00174	90.1976	0.00174	90.2225
RPM = 500.000 IDC = 4.00 FREQ OF INVERTER = 30.0 H				
1	0.67595	99.1472	0.67593	99.1501
5	0.01095	90.7347	0.01095	90.7383
7	0.00675	90.6260	0.00674	90.6365
11	0.00239	90.3445	0.00239	90.3550
13	0.00188	90.3122	0.00188	90.3297
17	0.00102	90.2159	0.00102	90.2337
19	0.00087	90.1976	0.00087	90.2225

CHAPTER 3

STEADY STATE ANALYSIS OF THREE PHASE INDUCTION MOTOR FED BY A CURRENT SOURCE INVERTER WITH A NONZERO COMMUTATION TIME

3.1 INTRODUCTION

This chapter presents the steady state analysis of the three phase squirrel cage induction motor fed by a current source inverter with a nonzero commutation time. It has bee shown in [1] that the rise or fall of the line currents of the motor during the commutation of the inverter can be appr mated by a function proportional to $I_{dc} \cos \omega_c t$, where ω_c is parameter dependent on the leakage inductance of the motor a the value of the commutating capacitor of the inverter. It also been shown in [1] that the time duration of the commuta- tion, denoted by t_c is given by

$$t_c = \pi/2\omega_c \tag{3.1}$$

The expressions for this assumed currents waveform have been given in Table 3.1. Fig. 3.1 shows the waveshapes of these line currents.

The dq model of the induction motor, with axis attached the stator (Sec. 1.1), is used for the analysis. The

transformed dq currents, using equations (4.39), for the three intervals of the inverter period are also listed in Table 3.1 and 'I' in Table 3.1, is defined by equation (2. The waveforms of the dq currents are given in the figure 3

The waveform of the rotor current is obtained by the solution of equations (1.6) and the phase voltages of th motor are obtained through equations (1.7).

The analysis has been done on the similar lines as th followed in Chapter 2. The difference in this case being different expressions of the line currents. Sections 3.2 3.5 correspond to the Sections 2.2 to 2.5 respectively. Section 3.2 deals with the time domain analysis, considerir both the stationary and the rotating rotor cases separately Section 3.3 deals with the frequency domain analysis and in Section 3.4 the voltage expressions at the stator terminals have been obtained. In Section 3.5 the case of the inducti motor introduced in Section 2.5 has been considered.

In this chapter, we have an additional parameter ω_c. the value of ω_c increases, the current waveshape tends to th ideal quasi square waveshape, corresponding to Fig. 2.1. It has also been shown in this chapter that as ω_c tends towards infinity, the rotor currents and stator voltage expressions this chapter approach those obtained in Chapter 2.

3.2 ANALYSIS OF INDUCTION MOTOR FOR ROTOR CURRENTS THROUGH TIME DOMAIN

The analysis proceeds in similar lines as done in Sec. The equation of the motor performance (1.6) is solved. The are solved for three intervals (I,II,III) only as the stato waveform is symmetric about $\omega t = \pi$. Each interval is consi red in two portions. One corresponds to the commutation pe and the other to the remaining interval period. So the fir interval corresponds to the time from start of interval, t_i less then time of commutation, t_c. This analysis has bee done in [12]. The results are reproduced here for all the three intervals. It should be noted that the symbols of ini tial currents correspond to equation (2.2).

3.2.1 Case of stationary rotor

The expressions of the rotor currents, as taken from [1 for intervals I,II and III have been given below.

3.7.1.1 Interval I $(o \leq \omega t \leq \pi/3)$

for $o \leq t \leq t_c$

$$i_{d2} = I_{do} \exp(-at) \tag{3.2}$$

$$i_{q2} = (I_{qo}+\sqrt{3}K_1) \exp(-at) - \sqrt{3}K_1[\cos(\omega_c t)-C_2 \sin(\omega_c t)] \tag{3.3}$$

where

$$I_{do} = \frac{3}{2} \frac{K_2 K_1}{C_4} \tag{3.4}$$

$$I_{qo} = \frac{-\sqrt{3}K_2K_1}{C_4} [\exp(-aT_c) - 1/2] \tag{3.5}$$

$$K_1 = M\omega_c^2 I/(a^2+\omega_c^2) L_{22} \tag{3.6}$$

$$C_2 = a/\omega_c \tag{3.7}$$

$$K_2 = [1+C_2 \exp(at_c)] \exp(-aT_c) \tag{3.8}$$

$$T_c = \text{interval period} = \pi/3\omega \tag{3.9}$$

and 'a', 'C_4' are as defined in equations (2.15) and (2.23) respectively.

for $t_c \leq t \leq T_c = \pi/3\omega$

$$i_{d2} = I_{do} \exp(-at) \tag{3.10}$$

$$i_{q2} = I'_{q2} \exp(-a(t-t_c)) \tag{3.11}$$

where

$$I'_{q2} = (I_{qo}+\sqrt{3}K_1) \exp(-at_c) + \sqrt{3}K_1C_2 \tag{3.12}$$

3.2.1.2 Interval II ($\frac{\pi}{3} \leq \omega t \leq \frac{2\pi}{3}$ i.e. $o \leq \omega t_2 \leq \frac{\pi}{3}$)

for $o \leq t < t_c$

$$i_{d2} = (I_{do} - \frac{3K_1}{2}) \exp(-at_2)+ \frac{3}{2} K_1[\cos(\omega_c t_2)-C_2 \sin(\omega_c t_2)] \tag{3.13}$$

$$i_{q2} = (I_{qo}+ \frac{\sqrt{3}K_1}{2}) \exp(-at_2) - \frac{\sqrt{3}K_1}{2} [\cos(\omega_c t_2) - C_2 \sin(\omega_c t_2)] \tag{3.14}$$

where

$$I_{qo} = \frac{\sqrt{3}K_2K_1}{C_4} [1 - \frac{\exp(-aT_c)}{2}] \tag{3.15}$$

$$I_{do} = \frac{3K_2K_1}{2C_4} [\exp(-aT_c)] \tag{3.16}$$

for $t_c \leq t_2 \leq T_c$, i.e. $((\frac{\pi}{3} + t_c) \leq \omega t \leq \frac{2\pi}{3})$

$$i_{d2} = I'_{d2} \exp(-a(t_2 - t_c)) \tag{3.17}$$

$$i_{q2} = I'_{q2} \exp(-a(t_2-t_c)) \tag{3.18}$$

where

$$I'_{d2} = (I_{do} - \frac{3K_1}{2}) \exp(-at_c) - \frac{3K_1C_2}{2} \tag{3.19}$$

$$I'_{q2} = (I_{qo} + \frac{\sqrt{3}K_1}{2}) \exp(-at_c) + \frac{\sqrt{3}K_1C_2}{2} \tag{3.20}$$

3.2.1.3 Interval III $(\frac{2\pi}{3} \leq \omega t \leq \pi$ i.e. $o \leq \omega t_3 \leq \frac{\pi}{3})$

for $o \leq \omega t_3 < t_c$ i.e. $\frac{2\pi}{3} \leq \omega t \leq (\frac{2\pi}{3} + t_c)$

$$i_{d2} = (I_{do} - \frac{3}{2} K_1) \exp(-at_3) + \frac{3}{2} K_1[\cos(\omega_c t_3) - C_2 \sin(\omega_c t_3)] \tag{3.21}$$

$$i_{q2} = (I_{qo} - \sqrt{\tfrac{3}{2}}\, K_1)\ \exp(-at_3) + \sqrt{\tfrac{3}{2}}\, K_1[\cos(\omega_c t_3) - C_2 \sin(\omega_c t_3) \tag{3.22}$$

where

$$I_{do} = \frac{3}{2} \frac{K_2 K_1}{C_4} [\exp(-aT_c) - 1] \tag{3.23}$$

$$I_{qo} = \sqrt{\tfrac{3}{2}} \frac{K_2 K_1}{C_4} [\exp(-aT_c) + 1] \tag{3.24}$$

for $t_c \leq \omega t_3 \leq \frac{\pi}{3}$

$$i_{d2} = I'_{d2} \exp(-a(t_3 - t_c)) \tag{3.25}$$

$$i_{q2} = I'_{q2} \exp(-a(t_3 - t_c)) \tag{3.26}$$

where

$$I'_{d2} = (I_{do} - \frac{3}{2} K_1) \exp(-at_c) - \frac{3}{2} K_1 C_2 \tag{3.27}$$

$$I'_{q2} = (I_{qo} - \sqrt{\tfrac{3}{2}}\, K_1) \exp(-at_c) - \sqrt{\tfrac{3}{2}}\, K_1 C_2 \tag{3.28}$$

The expressions of the initial and final values of currents fo each intervals have been summarised in Table 3.2. It can be seen from the table that the expressions for the final values of currents in an interval are the same as the initial values of currents in next interval. Hence, the rotor currents are continuous.

3.2.2 Case of rotating rotor

The expressions for quasi rotor currents because of the stator waveform of Fig. 3.1, have reproduced in this section from [12]. Actual rotor currents can be computed from these quasi rotor currents by using equations (2.44) and (2.45). Th symbols used below correspond to equation (2.58).

3.2.2.1 Interval I $(o \leq \omega t \leq \frac{\pi}{3})$

for $o \leq t < t_c$

$$X_1(t) = \exp(-at) \quad [A_{o1} \sin(\omega_r t)+B_{o1} \cos(\omega_r t)] - K_4 +$$

$$+ K_5 a[\omega_c \sin(\omega_c t)+K_2 \cos(\omega_c t)]+K_5\omega_c[\omega_c \cos\omega_c t-K_2 \sin\omega$$

(3.29)

where

$$B_{o1} = X_{1C} + C_1 \tag{3.30}$$

$$C_1 = K_4 - K_5K_2a - K_5\omega_c^2 \tag{3.31}$$

$$A_{o1} = (\dot{X}_{1O}+aB_{o1}-K_5 a\omega_c^2+K_5K_2\omega_c^2)/\omega_r \tag{3.32}$$

$$K_2 = (\omega_r^2+a^2-\omega_c^2)/2a \tag{3.33}$$

$$K_3 = 1/(\omega_c^2+K_2^2) \tag{3.34}$$

$$K_4 = \frac{\sqrt{3}}{2} K_1 I \; (\sqrt{3}\omega_r a)/(a^2+\omega_r^2) \tag{3.35a}$$

$$K_1 = M_a/L_{22} \tag{3.35b}$$

$$K_5 = \sqrt{3}K_1 I K_3/2a \tag{3.36}$$

$$\dot{X}_{10} = \omega_r X_{20} - aX_{10} + \frac{\sqrt{3}}{2} K_1 I \tag{3.37}$$

and

$$X_2(t) = \exp(-at)[A_{o2}\sin(\omega_r t) + B_{o2}\cos(\omega_r t)] + K_6 - K_5\,\omega_r(\omega_c \sin\omega_c t + K_2 \cos(\omega_c t)] \tag{3.38}$$

where

$$B_{o2} = X_{20} + C_3 \tag{3.39}$$

$$C_3 = K_5 K_2 \omega_r - K_6 \tag{3.40}$$

$$A_{o2} = (\dot{X}_{20} + aB_{o2} + K_5\omega_r\omega_c^2)/\omega_r \tag{3.41}$$

$$K_6 = \sqrt{3}K_1 I(\omega_r - \sqrt{3}a)/2(a^2+\omega_r^2) \tag{3.42}$$

$$\dot{X}_{20} = -\omega_r X_{10} - aX_{20} - \frac{3}{2} K_1 I \tag{3.43}$$

For X_{20} and X_{10} expression is given by

$$\begin{bmatrix} X_{20} \\ X_{10} \end{bmatrix} = \begin{bmatrix} 1/2-(Y_4Y_2-Y_1Y_3) & -\frac{\sqrt{3}}{2}+(Y_3Y_2+Y_1Y_4) \\ \frac{\sqrt{3}}{2}-(Y_3Y_2+Y_4Y_1) & \frac{1}{2}-(Y_4Y_2-Y_1Y_3) \end{bmatrix}^{-1} \begin{bmatrix} Z_4 \\ Z_3 \end{bmatrix} \tag{3.44}$$

where

$$Z_1 = C_2Y_1+C_1Y_2-K_4+K_5a\omega_c-K_5\omega_cK_2 \tag{3.45}$$

$$Y_1 = \exp(-at_c)\ \sin\ \omega_r t_c \tag{3.46}$$

$$Y_2 = \exp(-at_c)\ \cos\ \omega_r t_c \tag{3.47}$$

$$Y_3 = \exp(-a(T_c-t_c))\ \sin[\omega_r(T_c-t_c)] \tag{3.48}$$

$$Y_4 = \exp[-a(T_c-t_c)]\ \cos[\omega_r(T_c-t_c)] \tag{3.49}$$

$$C_2 = [\sqrt{\tfrac{3}{2}}\ K_1I+aC_1-K_1a\omega_c^2+K_5K_2\omega_c^2]/\omega_r \tag{3.50}$$

$$Z_2 = C_4Y_1+C_3Y_2+K_6-K_5\omega_rC_2 \tag{3.51}$$

$$C_4 = [C_3a-\tfrac{3}{2}\ K_1I+K_5\omega_r\omega_c^2]/\omega_r \tag{3.52}$$

$$Z_3 = C_5Y_3+(Z_1+K_4)Y_4-K_4 \tag{3.53}$$

$$C_5 = [-\sqrt{\tfrac{3}{2}}\ K_1I+K_4a+\omega_rZ_2]/\omega_r \tag{3.54}$$

$$Z_4 = Y_3C_6+Y_4(Z_2-K_6)+K_6 \tag{3.55}$$

$$C_6 = [-Z_1\omega_r-\tfrac{3}{2}\ K_1I-K_6a)]/\omega_r \tag{3.56}$$

The order of calculations of the constants for calculation of X_{10} and X_{20} is K_1 to K_6, Y_1 to Y_4, C_1 to C_4, Z_1, Z_2, C_5, C_6, Z_3 and Z_4.

for $t_c \leq t \leq \frac{\pi}{3}$

$$X_1(t) = \exp[-a(t-t_c)]\ [C_{o1}\ \sin(\omega_r(t-t_c) + D_{o1}\ \cos(\omega_r(t-t_c)]-K \tag{3.57}$$

$$X_2(t) = \exp[-a(t-t_c)]\ [C_{o2}\ \sin(\omega_r(t-t_c))+D_{o2}\ \cos(\omega_r(t-t_c))+K_6 \tag{3.58}$$

where

$$D_{o1} = X_1' + K_4 \tag{3.59}$$

$$C_{o1} = [\dot{X}_1' + aD_{o1}]/\omega_r \tag{3.60}$$

$$\dot{X}_1' = X_{20}Y_1+X_{10}Y_2+Z_1 \tag{3.61}$$

$$\dot{X}_1' = \omega_r X_2' - aX_1' - \frac{\sqrt{3}}{2} K_1 I \tag{3.62}$$

$$X_2' = -X_{10}Y_1+X_{20}Y_2+Z_2 \tag{3.63}$$

$$D_{o2} = X_2' - K_6 \tag{3.64}$$

$$C_{o2} = [\dot{X}_2' + aD_{o2}]/\omega_r \tag{3.65}$$

$$\dot{X}_2' = -\omega_r X_1' - aX_2' - \frac{3}{2} K_1 I \tag{3.66}$$

3.2.2.2 Interval II (i.e. (i.e. $\frac{\pi}{3} \leq t \leq \frac{2\pi}{3}$ or $o \leq \omega t_2 \leq \frac{\pi}{3}$)

for $o \leq t_2 < t_c$

$$X_1(t) = \exp(-at_2)\ [A_{o3}\ \sin(\omega_r t_2)+B_{o3}\ \cos(\omega_r t_2)] + -K_7+K_8[\omega_c\ \sin(\omega_c t_2)+K2\ \cos(\omega_c t_2)] + + K_9[\omega_c\ \cos(\omega_c t_2)-K_2\ \sin(\omega_c t_2)] \tag{3.67}$$

$$X_2(t) = \exp(-at_2)\ [A_{o4}\ \sin(\omega_r t_2)+B_{o4}\ \cos(\omega_r t_2)] + + K_{10}-K_{11}[\omega_c\ \sin(\omega_c t_2)+K_2\ \cos(\omega_c t_2)] + - K_{12}[\omega_c\ \cos(\omega_c t_2) - K_2\ \sin(\omega_c t_2)] \tag{3.68}$$

where

$$B_{o2} = X_{10} + K_7 - K_8K_2 - K_9\omega_c \tag{3.69}$$

$$A_{o3} = [\dot{X}_{10}+aB_{o3} -K_8\omega_c^2+K_9K_2\omega_c]/\omega_r \tag{3.70}$$

$$\dot{X}_{10} = \omega_r X_{20} - aX_{10} - \sqrt{\tfrac{3}{2}}\ K_1 I \tag{3.71}$$

$$K_7 = (\sqrt{3}IK_1a)/(a^2+\omega_r^2) \tag{3.72}$$

$$K_8 = [(\sqrt{3}I\ K_1a/2)-(3I\ K_1\ \omega_r/2)]\ (K_3/2a) \tag{3.73}$$

$$K_9 = \sqrt{3}I\ K_1\omega_c\ K_3/4 \tag{3.74}$$

$$B_{o4} = X_{20}-K_{10}+K_{11}K_2-K_{12}\omega_c \tag{3.75}$$

$$A_{o4} = [\dot{X}_{20}+aB_{o4}+K_{11}\omega_c^2-K_{12}K_2\omega_c]/\omega_r \tag{3.76}$$

$$\dot{X}_{20} = -\omega_r X_{10} - aX_{20} - \frac{3}{2} K_1 I \tag{3.77}$$

$$K_{10} = \sqrt{3}I \; K_1\omega_r/(a^2+\omega_r^2) \tag{3.78}$$

$$K_{11} = [(3K_1 \; Ia/2)+(\sqrt{3}K_1\omega_r \; I/2)](K_3/2a) \tag{3.79}$$

$$K_{12} = 3I \; K_1\omega_c \; K_3/4a \tag{3.80}$$

The values of X_{10} and X_{20} is determined from matrix equation

$$\begin{bmatrix} X_{20} \\ \\ X_{10} \end{bmatrix} = \begin{bmatrix} \frac{1}{2} - (Y_4Y_2-Y_1Y_3) & -\frac{\sqrt{3}}{2} + (Y_3Y_2+Y_4Y_1) \\ \\ \frac{\sqrt{3}}{2} - (Y_3Y_2+Y_4Y_1) & \frac{1}{2} - (Y_4Y_2-Y_1Y_3) \end{bmatrix}^{-1} \begin{bmatrix} Z_8 \\ \\ Z_7 \end{bmatrix} \tag{3.81}$$

The constants are to be solved in following order K_1 to K_3, K_7 to K_{12}, Y_1 to Y_4, C_7 to C_{10}, Z_5, Z_6, C_{11}, C_{12}, Z_7 and Z_8 where

$$C_7 = K_7 - K_8K_2 - K_9\omega_c \tag{3.82}$$

$$C_8 = [aC_7-(\sqrt{3}K_1 \; I/2)-K_8\omega_c^2+K_9K_2\omega_c]/\omega_r \tag{3.83}$$

$$C_9 = -K_{10}+K_{11}K_2+K_{12}\omega_c \quad (3.84)$$

$$C_{10} = [-(-3K_1\ I/2)+aC_9+K_{11}\omega_c^2-K_{12}K_2\omega_c]/\omega_r \quad (3.85)$$

$$Z_5 = C_8Y_1+C_7Y_2-K_7+K_8\omega_c-K_9K_{12} \quad (3.86)$$

$$Z_6 = C_{10}Y_1+C_9Y_2+K_{10}-K_{11}\omega_c+K_{12}K_2 \quad (3.87)$$

$$C_{11} = [Z_6\omega_r - \sqrt{3}\ K_1I+K_7a]\omega_r \quad (3.88)$$

$$C_{12} = [-Z_5\omega_r-K_{10}a]/\omega_r \quad (3.89)$$

$$Z_7 = C_{11}Y_3+(Z_5+K_7)Y_4-K_7 \quad (3.90)$$

$$Z_8 = C_{12}Y_3+(Z_6-K_{10})Y_4+K_{10} \quad (3.91)$$

For $t_c \leq \omega t_2 \leq \frac{\pi}{3}$

$$X_1(t_2) = \exp(-a(t_2-t_c))\ [C_{o3}\ \sin(\omega_r(\omega_r(t_2-t_c)) + $$
$$+ D_{o3}\ \cos(\omega_r(t_2-t_c))]-K_7 \quad (3.92)$$

$$X_2(t_2) = \exp(-a(t_2-t_c))\ [C_{o4}\ \sin\ \omega_r(t_2-t_c)) + $$
$$+ D_{o4}\ \cos(\omega_r(t_2-t_c))] + K_{10} \quad (3.93)$$

where

$$D_{o3} = X_1' + K_7 \quad (3.94)$$

$$C_{o3} = [\dot{X}_1' + aD_{o3}] \tag{3.95}$$

$$D_{o4} = X_2' - K_{10} \tag{3.96}$$

$$C_{o4} = [\dot{X}_2' + aD_{o4}]/\omega_r \tag{3.97}$$

X_1' and X_2' can be obtained from equations (3.67) and (3.68) by substituting $t_2 = t_c$. $\dot{X}_1'$ and $\dot{X}_2'$ can be obtained from equatio (2.50) and (2.51).

3.2.2.3 Interval III (i.e. $\frac{2\pi}{3} \leq \omega t \leq \pi$ or $o \leq \omega t_3 \leq \frac{\pi}{3}$)

for $o \leq t \leq t_c$

$$X_1(t) = \exp(-at_c)\,[A_{o5}\sin(\omega_r t_3)+B_{o5}\cos(\omega_r t_3)] +$$
$$+ K_{13}-K_{14}[\omega_c \sin(\omega_c t_3)+K_2\cos(\omega_c t_3)] +$$
$$- K_{15}[\omega_c \cos(\omega_c t_3) - K_2 \sin(\omega_c t_3)] \tag{3.98}$$

$$X_2(t) = \exp(-at_c)\,[A_{o6}\sin(\omega_r t_3)+B_{o6}\cos(\omega_r t_3)] +$$
$$+ K_{16}+K_{17}[\omega_c \sin(\omega_c t_3)+K_2\cos(\omega_c t_3)] +$$
$$- \sqrt{3}\,K_{15}[\omega_c \cos(\omega_c t_3) + K_2 \sin(\omega_c t_3)] \tag{3.99}$$

where

$$B_{o5} = X_{10} - K_{13} + K_{14}K_2 + K_{15}\,\omega_c \tag{3.100}$$

$$A_{o5} = [\dot{X}_{10}+aB_{o5} + K_{14}\omega_c^2-K_{15}K_2\omega_c]/\omega_r \qquad (3.101)$$

$$\dot{X}_{10} = \omega_r X_{20} - aX_{10} - K_1\sqrt{3}I \qquad (3.102)$$

$$K_{13} = [\sqrt{3}K_1 I/2]\ (\sqrt{3}\omega_r-a)/(a^2+\omega_r^2) \qquad (3.103)$$

$$K_{14} = [(\sqrt{3}K_1 I\ a/2)+(3K_1 I\omega_r/2)]\ (K_3/2a) \qquad (3.104)$$

$$K_{15} = \sqrt{3}K_1 I\omega_c K_3/4a \qquad (3.105)$$

$$B_{o6} = X_{20}-K_{16}-K_{17}K_2+\sqrt{3}K_{15}\omega_c \qquad (3.106)$$

$$A_{o6} = [\dot{X}_{20}+aB_{o6}-K_{17}\omega_c^2-\sqrt{3}K_{15}K_2\omega_c]/\omega_r \qquad (3.107)$$

$$\dot{X}_{20} = -\omega_r\ X_{10} - aX_{20} \qquad (3.108)$$

$$K_{16} = (\sqrt{3}K_2 I/2)(\omega_r+\sqrt{3}a)/(a^2+\omega_r^2) \qquad (3.109)$$

$$K_{17} = [(\sqrt{3}K_1\omega_r\ I/2)-(3K_1 aI/2)](K_3/2a) \qquad (3.111)$$

For the values of X_{20} and X_{10}, we have

$$\begin{bmatrix} X_{20} \\ X_{10} \end{bmatrix} = \begin{bmatrix} \frac{1}{2} - (Y_4Y_2-Y_1Y_3) & -\frac{\sqrt{3}}{2} + (Y_3Y_2+Y_4Y_1) \\ \frac{\sqrt{3}}{2} - (Y_3Y_2+Y_4Y_1) & \frac{1}{2} - (Y_4Y_2-Y_1Y_3) \end{bmatrix}^{-1} \begin{bmatrix} Z_{12} \\ Z_{11} \end{bmatrix} \qquad (3.111)$$

The constants are to be evaluated in the following order K_1 to K_3, K_{13} to K_{17}, Y_1 to Y_4, C_{13} to C_{16}, Z_9, Z_{10}, C_{17}, C_{18}, Z_{11} and Z_{12}, where

$$C_{13} = -K_{13}+K_{14}K_2+K_{15}\omega_c \tag{3.112}$$

$$C_{14} = [-\sqrt{3}K_1I+aC_{13}+K_{14}\omega_c^2-K_{15}K_2\omega_c]/\omega_r \tag{3.113}$$

$$C_{15} = -K_{16}-K_{17}K_2+\sqrt{3}K_{15}\omega_c \tag{3.114}$$

$$C_{16} = [aC_{15}-K_{17}\omega_c^2-\sqrt{3}K_{15}K_2\omega_c]/\omega_r \tag{3.115}$$

$$Z_9 = C_{14}Y_1+C_{13}Y_2+K_{13}-K_{14}\omega_c+K_{15}K_2 \tag{3.116}$$

$$Z_{10} = C_{16}Y_1+C_{15}Y_2+K_{16}+K_{17}\omega_c+\sqrt{3}K_{15}K_2 \tag{3.117}$$

$$C_{17} = [Z_{10}\omega_r-(\sqrt{3}K_1I/2)-K_{13}a]/\omega_r \tag{3.118}$$

$$C_{18} = [-Z_9\omega_r+(3K_1I/2)-K_{16}a]/\omega_r \tag{3.119}$$

$$Z_{11} = C_{17}Y_3+Y_4(Z_9-K_{13})+K_{13} \tag{3.120}$$

$$Z_{12} = C_{18}Y_3+(Z_{10}-K_{16})Y_4+K_{16} \tag{3.121}$$

For $t_c \leq t_3 \leq \frac{\pi}{3}$

$$X_1(t) = \exp(-a(t_3-t_c))[C_{o5}\sin(\omega_r(t_3-t_c)+D_{o5}\,\omega_r(t_3-t_c))]+K_{13} \tag{3.122}$$

$$X_2(t) = \exp(-a(t_3-t_c)[C_{o6} \sin(\omega_r(t_3-t_c))+D_{o6} \cos \omega_r(t_3-t_c))]+K \quad (3.123$$

where

$$D_{o5} = X_1' - K_{13} \quad (3.124$$

$$C_{o5} = [\dot{X}_1' + aD_{o5}]/\omega_r \quad (3.125$$

$$D_{o6} = X_2' - K_{16} \quad (3.126$$

$$C_{o6} = [\dot{X}_2' + aD_{o6}]/\omega_r \quad (3.127$$

The expressions for X_1' and X_2' can be obtained from equations (3.98) and (3.99) respectively by substituting $t_3 = t_c$. $\dot{X}_1'$ and $\dot{X}_2'$ can be obtained from equations (2.50) and (2.51) by substituting for X_1' and X_2'. A computer program has been develope to calculate initial and final values of currents for these intervals in the rotating rotor case and the boundary conditions are found to be matching (Sec. 3.5).

3.2.3 Case of $\omega_c \rightarrow \infty$

As ω_c increases, the stator current waveform of this chapter, (Fig. 3.1), tends towards the waveform considered in Chapter 2, (Fig. 2.1). In this section it has been shown that as ω_c tends towards infinity, the expressions for rotor currents obtained in Sec. 3.2.1 and 3.2.2 reduce to that obtained in Secs. 2.2.1 and 2.2.2. This reduction is shown

for interval I for both the sections. The results for other intervals can be shown by proceeding on similar lines.

3.2.3.1 Stationary rotor case

As $\omega_c \to \infty$ we note from equation (3.1)

$$t_c = 0 \tag{3.128}$$

So the interval $o \leq t < t_c$ of Sec. 3.2.1.1 reduce to a point $t = o$ and the period $t_c < t < T_c$ to the remaining period of interval I except $t = o$, as is the case in Sec. 2.2.1.1.

Also, as $\omega_c \longrightarrow \infty$

$$K_1 = \frac{MI}{L_{22}} = K_1' \quad \text{[From (3.6), (2.13)]} \tag{3.129}$$

$$C_2 = 0 \qquad \text{[From (3.7)]} \tag{3.130}$$

$$K_2 = \exp(-aT_c) = K_2' \quad \text{[From (3.8),(2.23),(3.130)]} \tag{3.131}$$

Thus from (3.129) and (3.131) I_{do} in equation (3.4) reduce to that in equation (2.21) and I_{qo} of equation (3.5) to that in equation (2.22). Also I'_{q2} of equation (3.12) reduce to $I'_{q2} = (I_{qo} + \sqrt{3}K_1)$ and this equation is similar to (2.12).

The expressions of currents in Sec. 3.2.1.1 become

$$i_{d2} = I_{do} \exp(-at) \qquad \text{[From eqn. (3.10)]} \tag{3.132}$$

$$i_{q2} = I'_{q2} \exp(-at) \qquad \text{[From eqn. (3.11)]} \tag{3.133}$$

These expressions are equivalent to equations (2.16) and (2.17) as I'_{q2} of (3.133) is already shown to reduce to that in (2.12) and I'_{q2} is I_{do} from eqn. (2.10) in equation (2.16). Thus the expressions obtained in 3.2.1.1 reduce to that obtained in 2.2.1.1 as ω_c approaches infinity.

3.2.3.2 Rotating rotor case

Here it will be shown that for ω_c approaching infinity, the rotor currents expressions of section 3.2.2.1 reduce to that obtained in Section 2.2.2.1. It has already been shown in 3.2.3.1 that sub regions of the interval taken in 3.2.2.1 reduce to that taken in 2.2.2.1 as $\omega_c \to \infty$.

<u>For t = o</u>

So, eqn. (3.29) is for a point t = o. Thus from (3.29)

$$X_1(t=o) = B_{o1} - K_4 + K_5 a K_2 + K_s \omega_c^2 \qquad (3.134)$$

Substituting for B_{o1} from (3.30) in (3.134)

$$X_1(t=o) = X_{1o} + C_1 - K_4 + K_5 K_2 a + K_5 \omega_c^2 \qquad (3.135)$$

Substituting for C_1 from (3.31) in (3.135)

$$X_1(t=o) = X_{1o} \qquad (3.136)$$

as should be

Similarly eqn. (3.38) reduce to

$$X_2(t=o) = X_{2o}\ .$$

<u>For X_{10} and X_{20}</u>

As $\omega_c \to \infty$, since $t_c = 0$, we have

$Y_1 = 0$ [from (3.46)] (3.136)

$Y_2 = 1$ [from (3.47)] (3.137)

$Y_3 = \exp(-aT_c) \sin \omega_r T_c$ [from (3.48)] (3.138)

$Y_4 = \exp(-aT_c) \cos \omega_r T_c$ [from (3.49)] (3.139)

$C_1 = K_4$ [from (3.31), (3.33), (3.34), (3.36)] (3.140)

$Z_1 = 0$ [from (3.45),(3.36),(3.34)] (3.141)

$C_2 = [-\sqrt{\frac{3}{2}} K_1 I + aK_4]/\omega_r$ [from (3.50)] (3.142)

$C_3 = -K_6$ [from (3.40)] (3.143)

$C_4 = [-K_6 a - \frac{3}{2} K_1 I]/\omega_r$ [from (3.52)] (3.144)

$Z_2 = 0$ [from (3.51),(3.136) to (3.144)] (3.145)

$C_5 = [-\sqrt{\frac{3}{2}} K_1 I + K_4 a]/\omega_r$ [from (3.55)] (3.146)

$C_6 = [-\frac{3}{2} K_1 I - K_6 a]/\omega_r$ [from (3.56)] (3.147)

$Z_3 = C_5 Y_3 + K_4 Y_4 - K_4$ [from (3.53),(3.141)] (3.148)

$Z_4 = Y_3 C_6 - Y_4 K_6 + K_6$ [from (3.56),(3.145)] (3.149)

and from (3.44) using (3.136) and (3.137)

$$\begin{bmatrix} X_{20} \\ X_{10} \end{bmatrix} = \begin{bmatrix} (Y_2 - Y_4) & -(\frac{\sqrt{3}}{2} - Y_3) \\ (\frac{\sqrt{3}}{2} - Y_3) & (\frac{1}{2} - Y_4) \end{bmatrix}^{-1} \begin{bmatrix} Z_4 \\ Z_3 \end{bmatrix} \qquad (3.150)$$

Thus it is observed that C_5 and C_6 of eqn. (3.146) and eqn. (3.147) correspond to $C_5^!$ and $C_6^!$ obtained in equations (2.72) and (2.75). Also Y_3 and Y_4 of Sec. 3.2.2.1 in reduced form for $\omega_c \to \infty$ in equations (3.138) and (3.139) correspond to Y_3 and Y_4 of Section 2.2.2.1, i.e., equations (2.68) and (2.69). Thus Z_3 and Z_4 from (3.148) and (3.149) are seen equivalent to $Z_3^!$ and $Z_4^!$ respectively, as in equations (2.71) and (2.74).

Thus equation (3.150) give the same value of X_{20} and X_{10} give the same value of X_{20} and X_{10} as obtained from equation (2.76), implying the similar initial conditions.

For t > o

The equations (3.57) and (3.58), using (3.128) can be seen to reduce as

$$X_1(t) = \exp(-at)\,[C_{o1} \sin \omega_r t + D_{o1} \cos \omega_r t] - K_4 \qquad (3.151)$$

$$X_2(t) = \exp(-at)\,[C_{o2} \sin \omega_r t + D_{o2} \cos \omega_r t] + K_6 \qquad (3.152)$$

It can also be shown from equations (3.59) to (3.66) that as $\omega_c \to \infty$

$$X_1' = X_{10} \tag{3.153}$$

$$X_2' = X_{20} \tag{3.154}$$

$$\dot{X}_1' = \omega_r X_{20} - aX_{10} - \frac{\sqrt{3}}{2} K_1 I \tag{3.155}$$

$$\dot{X}_2' = -\omega_r X_{10} - aX_{20} - \frac{3}{2} K_1 I \tag{3.156}$$

$$D_{o1} = X_{10} + K_4 \tag{3.157}$$

$$C_{o1} = [\dot{X}_1' + aD_{o1}]/\omega_r \tag{3.158}$$

$$D_{o2} = X_{20} - K_6 \tag{3.159}$$

$$C_{o2} = [\dot{X}_2' + aD_{o2}]/\omega_r \tag{3.160}$$

It is clear from equations (2.59), (2.60), (2.61), (2.66) and the above equations that

$$D_{o2}' = D_{o2} \tag{3.161}$$

and

$$C_{o2}' = C_{o2} \tag{3.162}$$

Thus equations (3.151) and (3.152) correspond to equations (2.56) and (2.62). Thus, as $\omega_c \to \infty$, the results obtained in Sec. 3.2.2.1 reduce to that obtained in Sec. 2.2.2.1, as it should be.

3.3 ANALYSIS OF INDUCTION MOTOR FOR ROTOR CURRENTS THROUGH FREQUENCY DOMAIN

The analysis to obtain the rotor currents through frequency domains has two steps, as is done in Sec. 2.3. First is the calculation of the harmonic components of the stator current. Then the harmonics of quasi rotor current, X_1 and X_2 are computed by the use of equations (2.52) and (2.53). The harmonics of actual rotor currents can be obtained from equations (2.42) and (2.43) after we have computed the harmonics of stator and quasi rotor currents.

For the harmonic components of the stator current, Fig.3.1, i_{d1} and i_{q1} can be written in Fourier series as

$$i_{d1} = \sum_{n=1}^{\infty} C_{n_d} \cos n\omega t + \sum_{n=1}^{\infty} d_{n_d} \sin n\omega t \tag{3.163}$$

$$i_{q1} = \sum_{n=1}^{\infty} C_{n_q} \cos n\omega t + \sum_{n=1}^{\infty} d_{n_q} \sin n\omega t \tag{3.164}$$

From Fig. 3.1, it can be said that for even values of n, the constants C_{n_d}, C_{n_q}, d_{n_d} and d_{n_q} are zero because of mirror image symmetry. For odd, values of n,

$$C_{n_d} = \frac{6I}{T} [-\sin \frac{n\pi}{3} \cdot \frac{2}{n\omega} - \frac{1}{(n\omega+\omega_c)} - \frac{1}{(n\omega-\omega_c)} + $$

$$+ \frac{1}{2} \frac{1}{(n\omega-\omega_c)} - \frac{1}{(n\omega+\omega_c)} \times \cos(\frac{n\pi\omega}{2\omega_c} + \frac{n\pi}{3})+\cos(\frac{n\pi\omega}{2\omega_c} + \frac{2\pi}{3})] \tag{3.165}$$

$$d_{n_d} = \frac{3I}{T}\left[\left(\frac{1}{(n\omega-\omega_c)} - \frac{1}{(n\omega+\omega_c)}\right) \times \sin\left(\frac{n\pi\omega}{2\omega_c} + \frac{n\pi}{3}\right) + \right.$$

$$\left. + \sin\left(\frac{n\pi\omega}{2\omega_c} + \frac{2n\pi}{3}\right)\right] \qquad (3.166)$$

$$C_{n_q} = -\frac{2}{\sqrt{3}} \sin \frac{n\pi}{3}\, d_{n_d} \qquad (3.167)$$

$$d_{n_q} = \frac{2}{\sqrt{3}} \sin \frac{n\pi}{3}\, C_{n_d} \qquad (3.168)$$

It can be seen from equations (3.165) to (3.168) that for n to be multiple of 3, these constants are zero. Thus from now, when we refer to harmonics we mean only odd and non triplents.

Thus, writing the mth harmonic component of i_{d1} and i_{q1} as

$$i_{d1m} = I'_{m_d} \cos(m\omega t + \beta_{m_d}) \qquad (3.169)$$

$$i_{q1m} = I'_{mq} \cos(m\omega t + \beta_{mq}) \qquad (3.170)$$

We note from equations (3.165) to (3.168) that

$$|I'_{m_d}| = |I'_{m_q}| = |I'_m| \qquad (3.171)$$

$$\beta_{m_q} = (\beta_{m_d} - \tfrac{\pi}{2}) \quad \text{for } m = 1,7,13,19 \ldots \qquad (3.172)$$

$$\beta_{m_q} = (\beta_{m_d} + \tfrac{\pi}{2}) \quad \text{for } m = 5,11,17,23, \ldots \qquad (3.173)$$

If we define 'p' as in equation (2.123) then

$$i_{d1m} = I'_m \cos(m\omega t + \beta_{m_d}) \tag{3.174}$$

$$i_{q1m} = I'_m \cos(m\omega t + \beta_{m_d} + (-)^p \ \frac{\pi}{2}) \tag{3.175}$$

where

$$I'_{m_d} = (C^2_{m_d} + d^2_{m_d})^{1/2} \tag{3.176}$$

$$\beta_{m_d} = \tan^{-1} (-d_{m_d}/C_{m_d}) \tag{3.177}$$

It should be remembered here, that m refers only to odd and non triplent value. Solving (2.52) and (2.53) for mth harmonic of X_1 and X_2 respectively, as done in Sec. 2.3 we can again visualise the picture as in Fig. 2.3. For this case we have

$$X_{1m} = - \frac{K_1 a(-1)^p}{A_T} I'_m \sin(m\omega t + \beta_{m_d} - \alpha_T) + $$

$$+ \frac{K_1 I'_m}{A_T} (\omega_r - (-1)^p \ m\omega) \cos(m\omega t + \beta_{m_d} - \alpha_T) \tag{3.178}$$

$$X_{2m} = - \frac{K_1 a I'_m}{A_T} \sin(m\omega t + \beta_{m_d} - \alpha_T) + $$

$$+ \frac{K_1 I'_m}{A_T} (\omega_r - (-1)^p \ m\omega) \sin(m\omega t + \beta_{m_d} - \alpha_T) \tag{3.179}$$

where A_T and α_T are given by equations (2.129) and (2.130). It can be noted from equations (3.178) and (3.179) that the magnitude of harmonics X_{1m} and X_{2m} are equal and they have a phase difference of $\pi/2$. In Sec. 3.5, harmonics of X_1 and X_2 have been computed for various values of ω_c and ω_r for a case of induction motor.

Case of $\omega_c \to \infty$

As ω_c increases, the stator current waveform of this chapter (Fig. 3.1) tends towards the waveform considered in Chapter 2, (Fig. 2.1). So, in the limit of ω_c tending towards infinity, the result of this section tend to that obtained in Sec. 2.3, as is shown below

$$\underset{\omega_c \to \infty}{\mathrm{Lt}} \quad d_{n_d} = 0 \tag{3.180}$$

$$\underset{\omega_c \to \infty}{\mathrm{Lt}} \quad C_{n_d} = \frac{6I}{T} \times \frac{2}{n\omega} \cdot -\sin\frac{n\pi}{3} = -\frac{6I}{n\pi}\sin\frac{n\pi}{3} \tag{3.181}$$

i.e.

$$\underset{\omega_c \to \infty}{\mathrm{Lt}} \quad C_{n_d} = a_{n_d} \tag{3.182}$$

Thus, from equations (2.126), (3.176), (3.180) and (3.182)

$$\underset{\omega_c \to \infty}{\mathrm{Lt}} \quad I'_m = a_{n_d} = I_m \tag{3.183}$$

From equations (3.177) and (3.180)

$$\beta_{m_d} = 0 \tag{3.184}$$

Substituting for I'_m and β_{m_d} from equations (3.183) and (3.184) in (3.178) and (3.179) we see that these expressions of harmonics of quasi rotor currents reduce to that obtained in Sec. 2.3, equations (2.127) and (2.131).

3.4 VOLTAGE WAVEFORM AT THE STATOR TERMINALS

The expressions for the voltage waveform at the stator terminals for the stator dq currents as in (Fig. 3.2), can be obtained by proceeding similar to Sec. 2.4. For this analysis each interval is divided into three regions. They correspond to time ($t_i < t_c$), ($t_i = t_c$) and the remaining period of the interval. In this section the cases of the stationary and the rotating rotor have been dealt with separately.

3.4.1 For the stationary rotor

The equations for stators voltages for this case of stationary rotor have been computed in Sec. 2.4.1. They are equations (2.137) and (2.138), i.e.,

$$V_{d1} = r_1 i_{d1} - Ma\, i_{d2} + (L_{11} - \frac{M^2}{L_{22}}) p i_{d1} \tag{3.185}$$

$$V_{q1} = r_1 i_{q1} - Ma\, i_{q2} + (L_{11} - \frac{M^2}{L_{22}}) p i_{q1} \tag{3.186}$$

Since i_{d1} and i_{q1} are continuous, (Fig. 3.2), there is no impulse in this case.

<u>At $t_i = 0$</u>

The jump in V_{d1} and V_{q1} at $t_i = 0$, given by ΔV_{d1} and ΔV_{q1}, can be computed from (3.185) and (3.186)

$$\Delta V_{d1} = (L_{11} - \frac{M^2}{L_{22}}) \, [\Delta(pi_{d1})] \Big|_{t_i=o} \tag{3.187}$$

$$\Delta V_{q1} = (L_{11} - \frac{M^2}{L_{22}}) \, [\Delta(pi_{q1})] \Big|_{t_i=o} \tag{3.188}$$

Table 3.1 can be used to compute the changes in slopes at $t_i = o$. From Table 3.1 it can be shown

$$\Delta(pi_{d1}) \Big|_{t_i=o} = 0 \tag{3.189}$$

$$\Delta(pi_{q1}) \Big|_{t_i=o} = 0 \tag{3.190}$$

where $i = 1, 2, 3, \ldots$

and so V_{d1} and V_{q1} are continuous at $t_i = 0$.

<u>For $o < t_i < t_c$</u>

The equations (3.185) and (3.186) can be used to compute the expressions of the voltages. Table 3.4 summarises the references to the expressions of i_{d1}, i_{q1}, i_{d2} and i_{q2} which are required for the computation of voltages.

<u>At $t_i = t_c$</u>

It is seen from (Fig. 3.2) that there is sudden chance of slopes of i_{d1} and i_{q1} at this point. Hence, there is a jump in the voltage expressions at $t_i = t_c$, as can be seen from equations (3.187) and (3.188).

From Table 3.1, using $t_c = \frac{\pi}{2\omega_c}$ from equation (3.1), we have

$$\Delta(pi_{d1})\Big|_{(t=t_c)} = 0 \tag{3.191}$$

$$\Delta(pi_{q1})\Big|_{(t=t_c)} = \sqrt{3}I\omega_c \tag{3.192}$$

$$\Delta(pi_{d1})\Big|_{(t_2=t_c)} = -\frac{3I}{2}\omega_c \tag{3.193}$$

$$\Delta(pi_{q1})\Big|_{(t_2=t_c)} = \frac{\sqrt{3}I}{2}\omega_c \tag{3.194}$$

$$\Delta(pi_{d1})\Big|_{(t_3=t_c)} = -\frac{3I}{2}\omega_c \tag{3.195}$$

$$\Delta(pi_{q1})\Big|_{(t_3=t_c)} = -\frac{\sqrt{3}I}{2}\omega_c \tag{3.196}$$

Equations (3.191) to (3.196) can be used in equations (3.187) and (3.188) to compute the jumps in V_{d1} and V_{q1} at $t_i = t_c$.

For $t_c < t_i < T_c$

During this interval i_{d1} and i_{q1} are constant. Hence, equations (3.185) and (3.186) reduce to

$$V_{d1} = r_1 i_{d1} - Ma\, i_{d2} \tag{3.197}$$

$$V_{q1} = r_1 i_{q1} - Ma\, i_{q2} \tag{3.198}$$

Table 3.3 gives the references for the computation of the voltages.

3.4.2 For the rotating rotor

From equations (2.148) and (2.149), using equations (2.42), (2.43) to elliminate for i_{d2} and i_{q2} we have

$$V_{d1} = A_2 i_{d1} - MaX_2 - M\omega_r X_1 + A_1 p i_{d1} \tag{3.199}$$

$$V_{q1} = A_2 i_{q1} - MaX_1 + M\omega_r X_2 + A_1 p i_{q1} \tag{3.200}$$

where A_1 and A_2 are defined in equations (2.140) and (2.143) respectively.

Since i_{d1} and i_{q1} are continuous, Fig. 3.2, and X_1, X_2 are also continuous, equation (2.45) and (2.46), there are no impulse in the expressions of V_{d1} and V_{q1}.

<u>At $t_i = 0$</u>

It has been seen in Section 3.4.1 that at $t_i = 0$, the changes in 'pi_{d1}' and 'pi_{q1}' are zero and hence V_{d1} and V_{q1} is continuous at $t_i = o$.

<u>For $o < t_i < t_c$</u>

The expressions for X_1 and X_2 obtained in Sec. 3.2.2 together with Table 3.1 is used in equations (3.199) and (3.200) to obtain the expressions for voltages. Table 3.5 lists the references to equation numbers for X_1 and X_2.

<u>For $t_i = t_c$</u>

From equations (2.199) and (2.200) because i_{d1}, i_{q1}, X_1 an X_2 are continuous

$$V_{d1}(t_i=t_c) = A_1(pi_{d1})\Big|_{t_i=t_c} \tag{3.201}$$

$$V_{q1}(t_i=t_c) = A_1(pi_{q1})\Big|_{t_i=t_c} \tag{3.202}$$

The equations (3.191) to (3.196) can be used in equations (3.201) and 9(3.202) to obtain these discontinuity of voltage

<u>For $t_c < t_i < T_c$</u>

In this region i_{d1} and i_{q1} are constant and hence equations (2.199) and (2.200) reduce to

$$V_{d1} = A_2\, i_{d1} - MaX_2 - M\omega_r X_1 \tag{3.203}$$

$$V_{q1} = A_2\, i_{q1} - MaX_1 + M\omega_r\, X_2 \tag{3.204}$$

Table 3.4 lists the references to the equations which give the expressions for X_1, X_2, i_{d1} and i_{q1} during their region, and can be used to compute v_{d1} and v_{q1}.

3.5 CALCULATIONS FOR A KNOWN MACHINE PARAMETERS

Using the results of Sec. 3.2 and Sec. 3.3 two computer programs are developed to compute rotor currents. The listing are given in Appendix B.

Using these programs the data for Tables 3.5 and 3.6 are obtained. It is seen from Table 3.5 that the I_{do} and I_{qo} values need to computed only for one interval. Table 3.6 shows that the solutions obtained through both the programs using Sec. 3.3 and Sec. 3.4 results are similar. It is also seen from Table 3.6 that as ω_c increases, the results approach to that obtained in Table 2.6 for ideal inverter current.

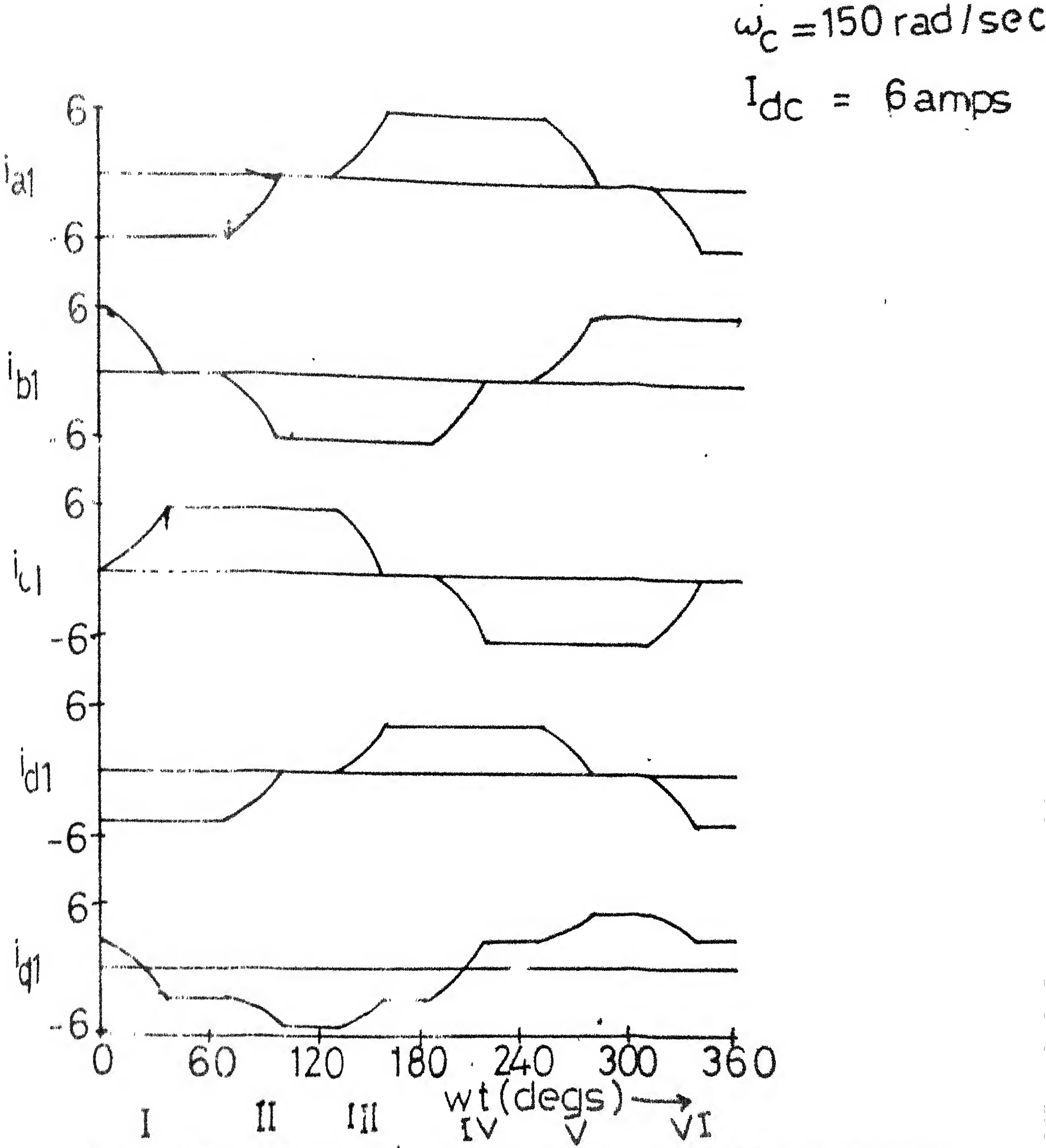


Fig 3.1 : Three phase & d-q stator currents

Table 3.1

Three phase and two phase stator currents

Interval/	3-phase currents		D-Q currents	
	During commutation	After commutation	During commutation	After commutation
I	$i_{a1}=-I_{dc}$	$-I_{dc}$	$I_{d1} = -3I/2$	$-3I/2$
$0 \le \omega t \le \pi/3$	$i_{b1}=I_{dc}\cos(\omega_c t)$	0	$i_{q1} = \sqrt{3}I(\cos(\omega_c t)-1/2)$	$-\sqrt{3}I/2$
	$i_{c1}=I_{dc}(1-\cos(\omega_c t))$	I_{dc}		
II	$i_{a1}=-I_{dc}\cos(\omega_c t_2)$	0	$i_{d1} = -\frac{3I}{2}\cos(\omega_c t_2)$	0
$\frac{\pi}{3} \le \omega t \le \frac{2\pi}{3}$	$i_{b1}=-I_{dc}(1-\cos(\omega_c t_2))$	$-I_{dc}$		
	$i_{c1}=I_{dc}$	I_{dc}	$i_{q1} = \sqrt{3}I(\frac{\cos(\omega_c t_2)}{2} - 1)$	$-\sqrt{3}I$
III	$i_{a1}=I_{dc}(1-\cos(\omega_c t_3))$	I_{dc}	$i_{d1}=\frac{3I}{2}(1-\cos(\omega_c t_3))$	$3I/2$
$\frac{2\pi}{3} \le \omega t \le \pi$	$i_{b1}=-I_{dc}$	$-I_{dc}$	$i_{q1} = -\frac{\sqrt{3}I}{2}(1+\cos(\omega_c t_3))$	$-\sqrt{3}I/2$
	$i_{c1}=I_{dc}\cos(\omega_c t_3)$	0		

Table 3.2

Initial and final values of D-Q rotor currents for stationary motor

Interval	I_{do}	I_{qo}	I''_{q2}	I''_{q2}
$0 \leq \omega t \leq \pi/3$ INTERVAL I	$\frac{3K_2K_1}{2C_4}$	$-\frac{\sqrt{3}K_2K_1}{C_4}[\exp(-aT_c)-1/2]$	$\frac{3K_2K_1}{2C_4}\exp(-aT_c)$	$\frac{\sqrt{3}K_2K_1}{C_4}[1-\exp(-aT_c)$
$\pi/3 \leq \omega t \leq 2\pi/3$ INTERVAL II	$\frac{3K_2K_1}{2C_4}\exp(-aT_c)$	$\frac{\sqrt{3}K_2K_1}{C_4}[1-\exp(-aT_c)]$	$\frac{3K_2K_1}{2C_4}[\exp(-aT_c)-1]$	$\frac{\sqrt{3}K_2K_1}{2C_4}[\exp(-aT_c)+1$
$2\pi/3 \leq \omega t \leq \pi$ INTERVAL III	$\frac{3K_2K_1}{2C_4}[\exp(-aT_c)-1]$	$\frac{\sqrt{3}K_2K_1}{2C_4}[\exp(-aT_c)-1]$	$-\frac{3K_2K_1}{2C_4}$	$\frac{\sqrt{3}K_2K_1}{C_4}[\exp(-aT_c)-1/$

Table 3.3

Reference for the computation of stator voltage expressions of S stationary rotor case

Interval region →	$o \leq t_i < t_c$			$t_i = t_c$		$t_c < t_i \leq T_c$		
Voltage expressions reference	Eqn. (3.185) and (3.186)			Eqn. (3.187) and (3.188)		Eqn. (3.197) and (3.198)		
Interval ↓	For i_{d1} and i_{q1}	for i_{d2}	for i_{q2}	for $\Delta(pi_{d1})$	for $\Delta(pi_{d1})$	for i_{d1} and i_{q1}	for i_{d2}	for i_{q2}
I	Table 3.1	eqn. (3.2)	eqn. (3.3)	eqn. (3.191)	eqn. (3.192)	Table 3.1	eqn. (3.10)	eqn. (3.11)
II	Table 3.1	eqn. (3.13)	eqn. (3.14)	eqn. (3.193)	eqn. (3.194)	Table 3.1	eqn. (3.17)	eqn. (3.18)
III	Table 3.1	eqn. (3.21)	eqn. (3.22)	eqn. (3.195)	eqn. (3.196)	Table 3.1	eqn. (3.25)	eqn. (3.26)

Table 3.4

Reference for the computation of stator voltage expressions of rotating rotor case

Interval region	$o \leq t_i < t_c$			$t_i = t_c$		$t_c < t_i \leq T_c$		
Voltage expressions reference	eqns.(3.199) and (3.200)			eqns.(3.201) and (3.202)		eqns.(3.203) and (3.204)		
Interval	for i_{d1} and i_{q1}	for X_1	for X_2	for (pi_{d1})	for (pi_{q1})	for i_{d1} and i_{q1}	For X_1	For X_2
I	Table 3.1	eqn. (3.29)	eqn. (3.38)	eqn.(3.191)	eqn. (3.192)	Table 3.1	eqn. (3.57)	eqn. (3.58)
II	Table 3.1	eqn. (3.67)	eqn. (3.68)	eqn.(3.193)	eqn. (3.194)	Table 3.1	eqn. (3.92)	eqn. (3.93)
III	Table 3.1	eqn. (3.98)	eqn. (3.99)	eqn.(3.195)	eqn. (3.196)	Table 3.1	eqn. (3.122)	eqn. (3.123)

TABLE 3.5
****

INTERVAL NUMBER	** FOR X1** INITIAL VALUE	FINAL VALUE	** FOR X2** INITIAL VALUE	FINAL VALUE
FOR IDC=6.AMP;INV. FREQ.=20.HZ;RPM=500.;WC=500.				
I	3.26250	0.69154	-1.09295	-3.37006
II	0.68468	-2.57277	-3.37185	-2.28387
III	-2.57780	-3.26243	-2.27890	1.09291
FOR IDC=8.AMP;INV. FREQ.=30.HZ;RPM=600.;WC=800.				
I	1.77978	0.92323	0.03512	-1.52152
II	0.92027	-0.85605	-1.52376	-1.56027
III	-0.85950	-1.77973	-1.55888	-0.03512
FOR IDC=4.AMP;INV. FREQ.=15.HZ;RPM=200.;WC=750.				
I	1.07492	0.41953	-0.13816	-0.99872
II	0.41779	-0.65515	-0.99998	-0.86266
III	-0.65713	-1.07490	-0.86181	0.13816
FOR IDC=6.AMP;INV. FREQ.=20.HZ;RPM=400.;WC=700.				
I	1.94711	0.74907	-0.26321	-1.81587
II	0.74558	-1.19805	-1.81783	-1.55661
III	-1.20152	-1.94707	-1.55462	0.26320

TABLE 3.6
****

-----FOR X1-----

FREQ.	VIA TIME DOMAIN MAGNITUDE	VIA TIME DOMAIN PHASE	VIA FREQ. DOMAIN MAGNITUDE	VIA FREQ. DOMAIN PHASE
FOR RPM = 100.;IDC = 6.AMP;INV. FREQ. =20.HZ & WC= 300.				
1	0.80476	73.32485	0.80476	73.32652
5	0.01918	-31.11028	0.01917	-31.10732
7	0.00740	-84.51469	0.00740	-84.54451
11	0.00105	134.53249	0.00106	134.53023
13	0.00068	42.42627	0.00068	42.55904
17	0.00038	261.08836	0.00038	260.81069
19	0.00025	191.31723	0.00025	191.13576
FOR RPM = 500.;IDC = 6.AMP;INV. FREQ. =20.HZ & WC= 300.				
1	3.41081	98.77277	3.41051	98.77671
5	0.01701	-31.27879	0.01698	-31.24336
7	0.00819	-84.35751	0.00820	-84.44720
11	0.00098	134.71308	0.00100	134.49919
13	0.00072	42.24791	0.00071	42.58531
17	0.00038	261.73957	0.00037	260.79729
19	0.00025	191.68243	0.00026	191.14775
FOR RPM = 500.;IDC = 8.AMP;INV. FREQ. =30.HZ & WC= 800.				
1	1.34658	85.64456	1.34655	85.64724
5	0.01983	22.83890	0.01983	22.87026
7	0.01108	-4.90470	0.01107	-4.92045
11	0.00289	-62.91205	0.00289	-62.90466
13	0.00182	266.12543	0.00182	266.05194
17	0.00056	194.26089	0.00056	193.78426
19	0.00035	148.15403	0.00035	148.23648
FOR RPM = 200.;IDC = 4.AMP;INV. FREQ. =15.HZ & WC= 1000.				
1	1.06012	99.04895	1.06010	99.05103
5	0.02199	64.46595	0.02200	64.48393
7	0.01286	53.38764	0.01285	53.38785
11	0.00447	31.00138	0.00448	31.06987
13	0.00335	20.04493	0.00335	20.04030
17	0.00170	-2.42762	0.00170	-2.29362
19	0.00137	-13.52101	0.00137	-13.53293
FOR RPM = 500.;IDC = 6.AMP;INV. FREQ. =20.HZ & WC=99999.				
1	3.45353	122.71660	3.45320	122.72068
5	0.02343	90.69155	0.02347	90.69465
7	0.01587	90.48001	0.01586	90.49350
11	0.00524	89.71415	0.00526	89.72614
13	0.00433	89.54817	0.00433	89.56972
17	0.00224	89.09412	0.00226	89.12122
19	0.00199	88.94177	0.00198	88.97070
FOR RPM = 500.;IDC = 6.AMP;INV. FREQ. =20.HZ & WC=9999999.				
1	3.45353	122.78787	3.45320	122.79196
5	0.02343	91.04879	0.02347	91.05105
7	0.01588	90.97852	0.01586	90.99247
11	0.00524	90.50157	0.00526	90.51206
13	0.00433	90.47295	0.00433	90.49623
17	0.00225	90.31304	0.00226	90.33283
19	0.00199	90.29401	0.00198	90.32490

TABLE 3.6
****

-----FOR X1-----

FREQ.	VIA TIME DOMAIN MAGNITUDE	VIA TIME DOMAIN PHASE	VIA FREQ. DOMAIN MAGNITUDE	VIA FREQ. DOMAIN PHASE
FOR RPM = 100.;IDC = 6.AMP;INV. FREQ. =20.HZ & WC= 300.				
1	0.80476	73.32485	0.80476	73.32652
5	0.01918	-31.11028	0.01917	-31.10732
7	0.00740	-84.51469	0.00740	-84.54451
11	0.00105	134.53249	0.00106	134.53023
13	0.00068	42.42627	0.00068	42.55904
17	0.00038	261.08836	0.00038	260.81069
19	0.00025	191.31723	0.00025	191.13576
FOR RPM = 500.;IDC = 6.AMP;INV. FREQ. =20.HZ & WC= 300.				
1	3.41081	98.77277	3.41051	98.77671
5	0.01701	-31.27879	0.01698	-31.24336
7	0.00819	-84.35751	0.00820	-84.44720
11	0.00098	134.71308	0.00100	134.49919
13	0.00072	42.24791	0.00071	42.58531
17	0.00038	261.73957	0.00037	260.79729
19	0.00025	191.68243	0.00026	191.14775
FOR RPM = 500.;IDC = 8.AMP;INV. FREQ. =30.HZ & WC= 800.				
1	1.34658	85.64456	1.34655	85.64724
5	0.01983	22.83890	0.01983	22.87026
7	0.01108	-4.90470	0.01107	-4.92045
11	0.00289	-62.91205	0.00289	-62.90466
13	0.00182	266.12543	0.00182	266.05194
17	0.00056	194.26089	0.00056	193.78426
19	0.00035	148.15403	0.00035	148.23648
FOR RPM = 200.;IDC = 4.AMP;INV. FREQ. =15.HZ & WC= 1000.				
1	1.06012	99.04895	1.06010	99.05103
5	0.02199	64.46595	0.02200	64.48393
7	0.01286	53.38764	0.01285	53.38785
11	0.00447	31.00138	0.00448	31.06987
13	0.00335	20.04493	0.00335	20.04030
17	0.00170	-2.42762	0.00170	-2.29362
19	0.00137	-13.52101	0.00137	-13.53293
FOR RPM = 500.;IDC = 6.AMP;INV. FREQ. =20.HZ & WC=99999.				
1	3.45353	122.71660	3.45320	122.72068
5	0.02343	90.69155	0.02347	90.69465
7	0.01587	90.48001	0.01586	90.49350
11	0.00524	89.71415	0.00526	89.72814
13	0.00433	89.54817	0.00433	89.56972
17	0.00224	89.09412	0.00226	89.12122
19	0.00199	88.94177	0.00198	88.97070
FOR RPM = 500.;IDC = 6.AMP;INV. FREQ. =20.HZ & WC=9999999.				
1	3.45353	122.78787	3.45320	122.79196
5	0.02343	91.04879	0.02347	91.05105
7	0.01588	90.97852	0.01586	90.99247
11	0.00524	90.50157	0.00526	90.51206
13	0.00434	90.47295	0.00433	90.49623
17	0.00225	90.31304	0.00226	90.33283
19	0.00199	90.29401	0.00198	90.32490

CHAPTER 4

COMPUTATION OF THE ELECTROMAGNETIC TORQUE OF THE CURRENT FED INDUCTION MOTOR

4.1 INTRODUCTION

The expression for the instantaneous electromagnetic torque in terms of dq stator and rotor currents is given by the equation (1.8). This equation, together with the expressions of dq stator and rotor currents, has been used to compute the torque produced by the squirrel cage induction motor fed by a three phase current source inverter.

In the previous chapters, both the time domain and the frequency domain expressions for the dq stator and rotor currents were obtained. The time domain expressions obtained for the dq stator and rotor currents can be directly used in the equation (1.8) to obtain the instantaneous torque waveform

The torque spectra can also be obtained using the harmonic component values of the stator and rotor currents in equation (1.8). The instantaneous torque expression is given by the terms, which are product of the stator and rotor instantaneous currents. Thus if a nth harmonic of the stator current and an mth harmonic of the rotor current, referred to the stator is assumed then these will produce (m+n)th and the (m-n)th harmoni

component of the torque. It has been shown in this chapter that the contribution to the torque is only because of one of these terms. Further it has been shown that the torque harmonic components are multiples of three for the case of ge ral stator current. For the case of motor fed by the curren source inverter, the torque harmonic frequencies are multiple of six times the inverter frequency.

A torque harmonic has contributions from the various combinations of the stator and rotor current harmonics. But as the frequency of the harmonic current increases the contri- bution of these to the torque harmonic goes on decreasing. In fact the contribution due to two dominant harmonics of curren is seen to aporoximate fairly well to the actual value of the torque harmonic.

In Section 4.2 the nature of the harmonic components of the torque in any arbitary reference frame is discussed. The torque harmonic frequencies are found to multiple of three times the fundamental frequency for the case of general stator current. The general expression for the torque produced due to nth harmonic of stator current and mth harmonic of rotor current, when referred to the rotor, has been computed in this section.

Section 4.3 gives the procedures for the computation of the torque. One uses the instantaneous stator and rotor

current expressions to obtain the electromagnetic torque produced. In the other the harmonic components of the torque have been computed using the harmonic component values of the stator and rotor current.

In Section 4.4 the stator current waveform of Fig. 2.1 ha been used to compute the torque produced, using both the procedures of Section 4.3. The harmonic components of the instar taneous torque waveform obtained are calculated. These are compared with the torque harmonic values obtained directly through the other procedure. This verifies that both are procedures compute the same result.

4.2 NATURE OF THE HARMONIC COMPONENTS IN THE TORQUE

In this section, the nature of the harmonic components in the torque produced by the squirrel cage induction motor has been established. To start with a arbitary stator current waveform is studied. The expression for torque harmonic produced due to the interaction of nth harmonic of stator current and mth harmonic of the rotor current, when referred to the stator, is computed, in a arbitary reference frame.

It turns out that torque harmonic is independent of the choice of the reference frame and the torque harmonics have frequencies which are multiples of three times the stator current frequency.

The stator current of the motor when fed by the current source inverter is symmetric about $\omega t = \pi$. Also it does not have triplens. The nature of the torque for such a case of the stator current is also discussed in this section. Torque harmonic frequencies are multiples of six times the inverter frequency in this case.

4.2.1 Torque Harmonic in an arbitary reference frame

The torque harmonic produced by the squirrel cage induction motor fed by a three phase periodic nonsinusoidal current of frequency ω, has been computed in this section. The stator current harmonics will have the frequencies of the type '$n\omega$', where n is a positive integer. The rotor currents referred to stator are computed after applying a transfer function (Fig. 2.2) to the stator currents. Thus the frequencies of the rotor currents referred to the stator are also of the type '$m\omega$'.

The torque produced due to interaction of a stator current of frequency '$n\omega$' and a rotor current of frequency '$m\omega$', when referred to the stator has been studied in this section. The rotor is assumed to be rotating with a constant speed ω_r, in the direction of field produced by fundamental stator current. Then the harmonics with value of 'm' of the type (3p-1) produce the flux, revolving in opposite direction to the rotor speed. Here, p = 1,2,3,4 ...

Thus the actual rotor currents will have the frequencies of the type $(m\omega \pm \omega_r)$ where positive sign is taken for the case where 'm' is of the type (3p-1), and negative otherwise.

The expressions for the stator and rotor currents with a arbitary values of magnitudes and phases, I_s, α_s and I_r, α_r respectively, can be written as

$$i_{a1} = I_s \cos[n\omega t - \alpha_s] \tag{4.1}$$

$$i_{b1} = I_s \cos[n(\omega t - \tfrac{2\pi}{3}) - \alpha_s] \tag{4.2}$$

$$i_{c1} = I_s \cos[n(\omega t - \tfrac{4\pi}{3}) - \alpha_s] \tag{4.3}$$

$$i_{a2} = I_r \cos[m\omega t \pm \omega_r t - \alpha_r] \tag{4.4}$$

$$i_{b2} = I_r \cos[m(\omega t - \tfrac{2\pi}{3}) \pm \omega_r t - \alpha_r] \tag{4.5}$$

$$i_{c2} = I_r \cos[m(\omega t - \tfrac{4\pi}{3}) \pm \omega_r t - \alpha_r] \tag{4.6}$$

where lower sign is for the case when m is of the type (3p-1). Upper sign is for all other cases,

The currents in an arbitary reference frame [3] are related to line currents $i_{a1}(t)$, $i_{a2}(t)$ etc., from equations (1.1) to (1.4), as

$$i_{d1} = (2/3)\ [i_{a1} \cos\theta + i_{b1} \cos(\theta - \tfrac{2\pi}{3}) + i_{c1} \cos(\theta + \tfrac{2\pi}{3})] \quad (4.7$$

$$i_{q1} = (2/3)\ [-i_{a1} \sin\theta - i_{b1} \sin(\theta - \tfrac{2\pi}{3}) - i_{c1} \sin(\theta + \tfrac{2\pi}{3})] \quad (4.8$$

$$i_{d2} = (2/3)\ [i_{a2} \cos\beta + i_{b2} \cos(\beta - \tfrac{2\pi}{3}) + i_{c2} \cos(\beta + \tfrac{2\pi}{3})] \quad (4.9$$

$$i_{q2} = (2/3)\ [-i_{a2} \sin\beta - i_{b2} \sin(\beta - \tfrac{2\pi}{3}) - i_{c2} \sin(\beta + \tfrac{2\pi}{3})] \quad (4.10$$

where

$$\beta = \theta - \theta_r \qquad (4.11$$

$$\theta_r = \omega_r t \qquad (4.12$$

In above ω_r is the constant rotor speed. $(d\theta/dt)$ is the speed of the arbitary reference frame.

The torque is related to dq currents from equation (1.8)as

$$T_q = M_c[i_{q1}\ i_{d2} - i_{d1}\ i_{q2}) \qquad (4.14)$$

where

$$M_c = M(m/2)(P/2) \qquad (4.15)$$

Here m is the number of phase and P the number of poles.

Substituting for dq currents in equation (4.14) from equation (4.7) to eqn. (4.10) and simplifying gives

$$T_q = (4/9)\, M_c[e_1(t)\sin(\beta-\theta) + e_2(t)\sin(\beta-\theta-\tfrac{2\pi}{3}) +$$

$$+ e_3(t)\sin(\beta-\theta+\tfrac{2\pi}{3})] \qquad (4.16)$$

where

$$e_1(t) = (i_{a1}\, i_{a2} + i_{b1}\, i_{b2} + i_{c1}\, i_{c2}) \qquad (4.17)$$

$$e_2(t) = (i_{a1}\, i_{b2} + i_{b1}\, i_{c2} + i_{c1}\, i_{a2}) \qquad (4.18)$$

$$e_3(t) = (i_{a2}\, i_{b1} + i_{b2}\, i_{c1} + i_{c2}\, i_{a1}) \qquad (4.19)$$

Substituting for $i_{a1}(t)$, $i_{a2}(t)$ in equation (4.17) from equations (4.1) to (4.6), gives

$$e_1(t) = I_sI_r[\cos(n\omega t-\alpha_s)\cos(m\omega t \pm \omega_r t - \alpha_r) +$$

$$+ \cos(n(\omega t-\tfrac{2\pi}{3})-\alpha_s)\cos(m(\omega t-\tfrac{2\pi}{3}) \pm \omega_r t - \alpha_r) +$$

$$+ \cos(n(\omega t-\tfrac{4\pi}{3})-\alpha_s)\cos(m(\omega t-\tfrac{4\pi}{3}) \pm \omega_r t-\alpha_r)]$$

This implies

$$e_1(t) = (I_sI_r/2)[\cos[(n+m)\omega t-(\alpha_s+\alpha_r)\pm\omega_r t]+$$

$$+ \cos[(n+m)(\omega t-\tfrac{2\pi}{3})\pm \omega_r t-(\alpha_s+\alpha_r)] + \cos[(n+m)(\omega t-\tfrac{4\pi}{3})$$

$$\pm \omega_r t-(\alpha_s+\alpha_r)] + \cos[(n-m)\omega t \mp \omega_r t-(\alpha_s-\alpha_r)] +$$

$$+ \cos[(n-m)(\omega t-\tfrac{2\pi}{3}) \mp \omega_r t-(\alpha_s-\alpha_r)] +$$

$$+ \cos[(n-m)(\omega t-\tfrac{4\pi}{3}) \mp \omega_r t-(\alpha_s-\alpha_r)]\,] \qquad (4.20)$$

This can be shown to simplify to

$$e_1(t) = (3I_sI_r/z)\ [p_s \cos[\,(n+m)\omega t \pm \omega_r t - (\alpha_s+\alpha_r)\,] +$$

$$+\ p_d \cos[\,(n-m)\omega t \mp \omega_r t - (\alpha_s-\alpha_r)\,]\,] \qquad (4.21)$$

where

$$p_s = 1 \quad \text{if } (n+m) \text{ is of type '3p'}$$

$$= 0 \qquad (4.22)$$

$$p_d = 1 \quad \text{if } (n-m) \text{ is of type '3p'}$$

$$= 0 \quad \text{otherwise} \qquad (4.23)$$

Here p is any integer.

Similarly from equations (4.18) and (4.19), it can be shown that

$$e_2(t) = (3I_sI_r/2)\ [p_s \cos[(n+m)\omega t - \tfrac{2\pi m}{3} \pm \omega_r t - (\alpha_s+\alpha_r)] +$$

$$+\ p_d \cos[(n-m)\omega t + \tfrac{2\pi m}{3} \mp \omega_r t - (\alpha_s-\alpha_r)]\,] \qquad (4.24)$$

$$e_3(t) = (3I_sI_r/2)\ [p_s \cos[(n+m)\omega t + \tfrac{2\pi m}{3} \pm \omega_r t - (\alpha_s+\alpha_r)] +$$

$$+\ p_d \cos[(n-m)\omega t - \tfrac{2\pi m}{3} \mp \omega_r t - (\alpha_s-\alpha_r)]\,] \qquad (4.25)$$

The expressions of $e_1(t)$, $e_2(t)$ and $e_3(t)$ from equations (4.21), (4.24) and (4.25) are substituted in equation (4.16). Table 4.1 lists the final expression of torque obtained after

simplification for various combinations of 'n' and 'm' values. In the Table 4.1, 'n' corresponds to stator current harmonic and 'm' the rotor current harmonic when referred to the stator and p is a positive integer, i.e. 0,1,2,3 ...

It can be seen from Table 4.1 that the torque spectrum has frequencies which are multiple of '3ω'.

The frequency components of the type (3p+2) produce the flux rotating in opposite direction to the flux produced by the fundamental. The different torque expressions written in Table 4.1 can be written as one general expression if the frequency component of type (3p+2) and its phase is assigned a negative value. This implies that m or n takes the values as

$$1,-2,3,4,-5,6,7,-8,9 \ldots \tag{4.26}$$

If this is done, the torque is given as

$$T_q = 2I_sI_rM_c \sin[(n-m)\omega t-(\alpha_s^*-\alpha_r^*)] \quad \text{if } (n-m) \text{ is multiple of } 3$$

$$= 0 \qquad \text{otherwise} \tag{4.27}$$

where n and m takes the values as given by equation (4.27) and

$$\alpha_s^* = \alpha_s \qquad \text{if n is positive}$$

$$= -\alpha_s \qquad \text{if n is negative} \tag{4.28}$$

$$\alpha_r^* = \alpha_r \qquad \text{if m is positive}$$

$$= -\alpha_r \qquad \text{if m is negative} \tag{4.29}$$

Thus if n and m are represented by values given by equation (4.27), it is seen that the torque harmonic given by the difference of the stator and rotor current frequencies is produced. The equation for the torque (4.28) can be seen to be independent of the choice of the reference frame.

4.2.2 Harmonic components of the torque for the induction motor fed by a current source inverter

The stator current of the induction motor fed by a current source inverter is antisymmetric about $\omega t = \pi$, i.e. $i(\theta) = -i(\theta + \pi)$. This implies the absence of the even harmonics in the stator current. The triplens are also absent because the sum of three phase current will be zero in this case. Thus the permissible values of stator harmonics from equation (4.27) are

$$1, -5, 7, -11, 13, -17, 19 \qquad (4.30)$$

In a induction motor rotating at constant speed the induced rotor currents, referred to the stator will also have harmonics given by equation (4.30). Thus stator and rotor currents have odd and nontriplens values only.

It has been shown in Sec. 4.2.1 that torque has contribution from the harmonics when (n-m) is triplen. Values of 'n' and 'm' in this case, equation (4.30), are odd, and thus (n-m) is even. Thus the torque spectrum of the induction motor

being fed by the current source inverter have harmonics which are multiplies of six.

In previous chapters, the harmonics of the dq frame stator and the rotor currents have been obtained. The nth harmonic and mth harmonic of stator and rotor d axis current can be written as

$$i_{d1} = I_1 \cos(n\omega t - \alpha_1) \tag{4.31}$$

$$i_{d2} = I_2 \cos(m\omega t - \alpha_2) \tag{4.32}$$

I_1, I_2, α_1 and α_2 can be related to I_s, I_r, α_s and α_r as follows. Substituting for the values for the values of current in equation (4.7) from equations (4.1) to (4.3).

$$i_{d1} = \frac{2}{3} I_s[\cos(n\omega t-\alpha_s) \cos\theta +$$

$$+ [\cos[n(\omega t - \frac{2\pi}{3})-\alpha_s)] \cos(\theta - \frac{2\pi}{3})] +$$

$$+ [\cos[n(\omega t - \frac{4\pi}{3})-\alpha_s)] \cos(\theta - \frac{4\pi}{3})]$$

Simplifying this, we obtain

$$i_{d1} = I_s \cos(n\omega t - \alpha_s \pm \theta) \tag{4.33}$$

where upper sign is when n is of type (6p-1) and lower otherwise.

Comparing equations (4.33) and (4.31) gives

$$I_s = I_1 \tag{4.34}$$

$$\alpha_s = \alpha_1 \pm \theta \tag{4.35}$$

Similarly it can sbe shown that

$$I_r = I_2 \tag{4.36}$$

$$\alpha_r = (\alpha_2 \pm \beta \pm \omega_r t) \tag{4.37}$$

$$= \alpha_2 \pm \theta \tag{4.37b}$$

Substituting from equations (4.34) to (4.37) in equation (4.28),

$$T_q = 2I_1 I_2 M_c \sin[(n-m)\omega t - (\alpha_1 - \alpha_2)] \tag{4.38}$$

Here n and m takes value as given in equation (4.31).

The signs of α_1 and α_2 are reversed if n or m respectively has a negative value.

It should be further noted that the harmonics of the pseudo rotor currents, X_1 and X_2, rather than rotor currents have been computed in the previous chapters. The dependence of torque on rotor currents can be changed to pseudo rotor currents by equations (2.42) and (2.43). Substituting these in equation (4.14) gives

$$T_q = M_c \left[i_{q1}\left(X_2 - \frac{M}{L_{22}} i_{d1}\right) - i_{d1}\left(X_1 - \frac{M}{L_{22}} i_{q1}\right)\right]$$

This implies that

$$T_q = M_c[i_{q1} X_2 - i_{d1} X_1] \tag{4.39}$$

It is seen that this equation (4.39) is similar to the equation (4.1) except for the change of i_{d2} and i_{q2} to X_2 and X_1. respectively. X_2 and X_1 can be seen as 'd' and q axis component from equations (2.42) and (2.43). Thus a equation similar to equation (4.38) can be obtained in terms of the harmonic components of the pseudo rotor currents as

$$T_q = 2I_1 X_r M_c \sin[(n-m)\omega t-(\alpha_1-\alpha_x)] \tag{4.40}$$

Here X_r is mth harmonic magnitude of X_2. α_x is its phase if 'm' is positive and negative of phase otherwise. 'm' take values as given in equation (4.31).

4.3 METHODS FOR THE COMPUTATION OF THE TORQUE

The expression relating the dq stator and rotor currents to the instantaneous electromagnetic torque is given by equation (4.1). In previous chapters, two methods for the computation of the rotor currents from a given stator current have been described.

The first method results in the time domain solution of the rotor current and the other gives the harmonic components of the rotor current.

A method to compute the torque produced by the squirrel cage induction is to substitute this time domain solution of the rotor current obtained, in equation (4.1) to get the time domain torque expression. This method is described in Sec. 4.3.1.

Other method uses the values of the harmonic components of stator and rotor current obtained. Equation (4.39) gives the expression for the harmonic torque produced due to the interaction of the nth harmonic of the stator and mth harmonic of the rotor current. For the system of motor being fed by the current source inverter, m or n takes values as given by the equation (4.31). For computation of particular torque harmonic component, which is multiple of six (Sec. 4.2.2), all the possible combinations of the permissible values of 'm' and 'n' are taken which contribute to this torque harmonic. Their contributions to the torque can be obtained from equation (4.38). In Section 4.3.2, the procedure to evaluate all the possible combinations for a particular harmonic has been mentioned.

4.3.1 Time domain solution of the torque

In previous chapters, the method for the time domain solution of rotor currents has been given. There the cases of stationary and rotating rotors have been dealt with separately. For the case of stationary rotor, the rotor currents expressions have been obtained. This can be substituted in

equation (4.14) to obtain the torque produced for the stationary rotor case.

For the case of rotating rotor the solutions obtained for X_1 and X_2 can be substituted in equation (4.40) to obtain the instantaneous torque values.

It has been shown in Section 4.2.2 that the torque has harmonics to be multiple of six times the inverter frequency. It implies that the torque waveform repeats after every 60^{o} interval. Thus the computation for this section has to be done only for any one of six intervals of the inverter.

4.3.2 Frequency domain solution of the torque

The value of a particular torque harmonic can be computed through equation (4.40). For this computation, first of all, the possible combination of the current harmonics of the stator and rotor have to be obtained which contribute to this torque harmonic.

The stator and rotor currents have infinite number of the harmonic components. The magnitudes of these harmonic go on decreasing as the number of harmonic increases. Thus though there are infinite number of the combination which will give the particular torque harmonic, the contribution because of higher order current harmonics will be very insignificant.

In the section the evaluations of stator and pseudo rotor currents harmonic combinations is done in a special sequence. This sequence has a property that at each step all the

permissible combination of harmonic currents upto a specific frequency are computed. The torque value is updated with every new combination taken till it is observed that the contribution because of coming new pairs to the torque is less than a limit (say 0.001%) of the last updated value.

This calculation of the torque harmonic has the following four steps.

(i) To get the initial pair of stator and rotor frequency to start the torque computation. This has been shown in Fig. 4.1.

(ii) To compute the torque value by a given pair of frequencies. This computation has been shown in Fig. 4.2.

(iii) To check when to terminate the present process of computation. This is done by checking the contribution of the last pair to the torque. Further calculations end if this contribution is less than a limit (say 0.001%) and the two possible combinations with a component of pair 1 had been considered. If this is not the case we move to next step.

(iv) This step computes the next permissible pair which contributes to the chosen torque harmonic.

The flow chart of the procedure for computation of the chosen torque harmonic has been given in Fig. 4.3.

4.4 CALCULATIONS FOR A KNOWN MACHINE PARAMETERS

A computer program is developed to compute torque harmonic using the flow chart of Fig. 4.3. The time domain torque is obtained via Sec. 4.3.1 approach. The listing of computer program is given in the Appendix B.

The average and sixth harmonic torque of induction motor produced by the stator current of Fig. 2.1 has been plotted in Fig. 4.4 as the function of rotor speed. It is seen that sixth harmonic torque is maximum at synchronous speed.

Fig. 4.5 gives the relative contents of various torque harmonics at three rotor speed, It can be noted from this figure that as the rotor speed is increased the torque harmonic components become more and more dominant.

Fig. 4.6 plots the time domain torque waveform. This waveform has been drawn for 60^{o} interval only because the torque waveform repeats after every 60^{o} interval (Sec. 4.2).

Table 4.2 compares the values of torque harmonics obtained from following methods :

(a) Using Sec. 4.3 method of obtaining torque harmonics from harmonics of currents.

(b) Obtaining the harmonic components of the time domain solution of the torque, obtained through Sec. 4.2 approach.

(c) Obtaining the value of torque harmonic by considering the dominant current harmonics only. For the torque harmonic frequency of 6 nω, the dominant current harmonics have frequency $(6n-1)\omega$ and $(6n+1)\omega$ because these interact with the fundamental to produce the harmonic torque.

It is seen from Table 4.1 that all the three methods give the similar results.

START

Divide the nth torque harmonic component value into two halves, n/2 and n/2

Go on incrementing one value and decrementing the other till neither is even or triplen

The number corresponding to type (6p-1) is assigned the negative value

The two numbers form the initial pair of frequencies

END

Fig. 4.1 : Step 1 Computation of the initial starting pair of frequency of the stator and rotor current

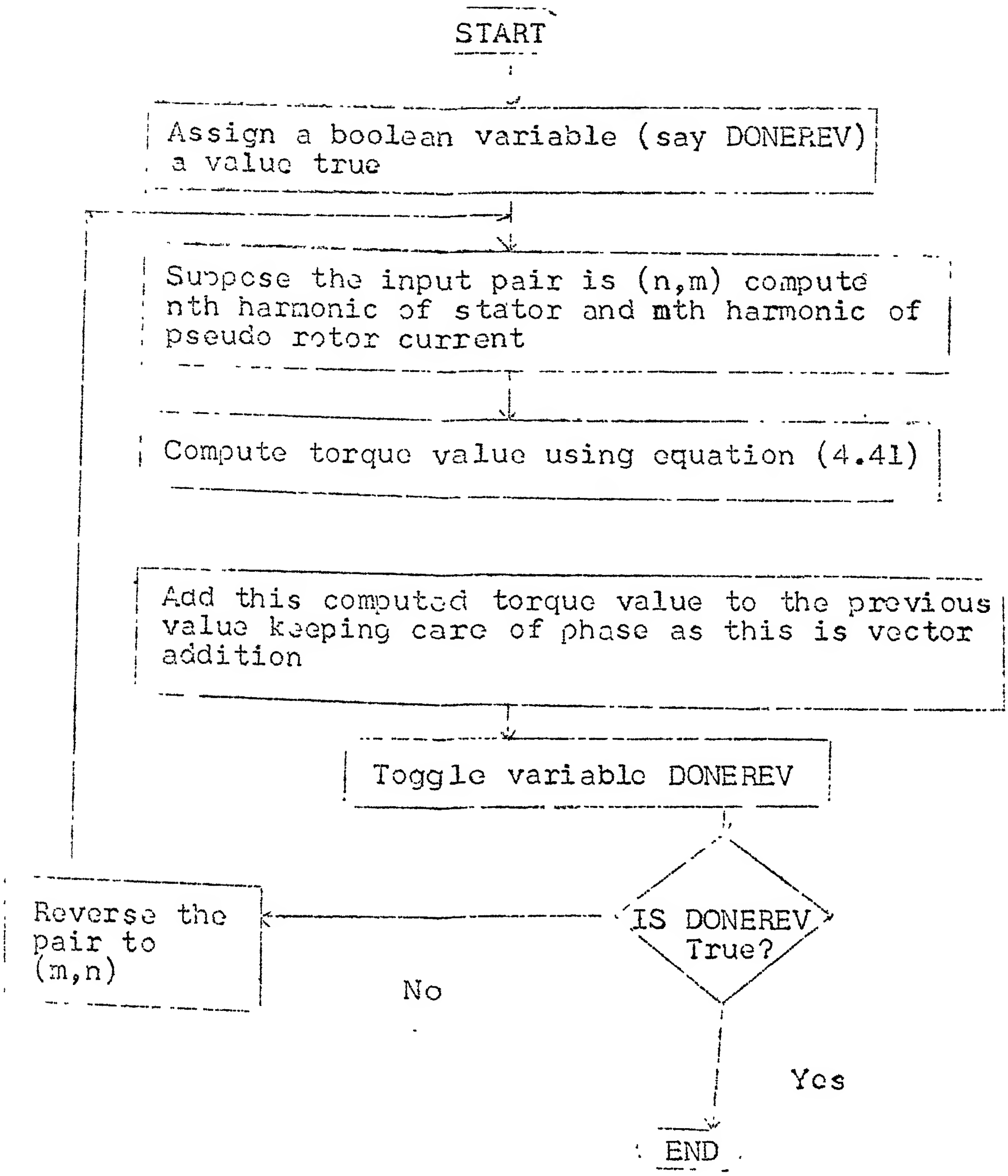


Fig. 4.2 : Step 2 Updating of torque with the new pair value

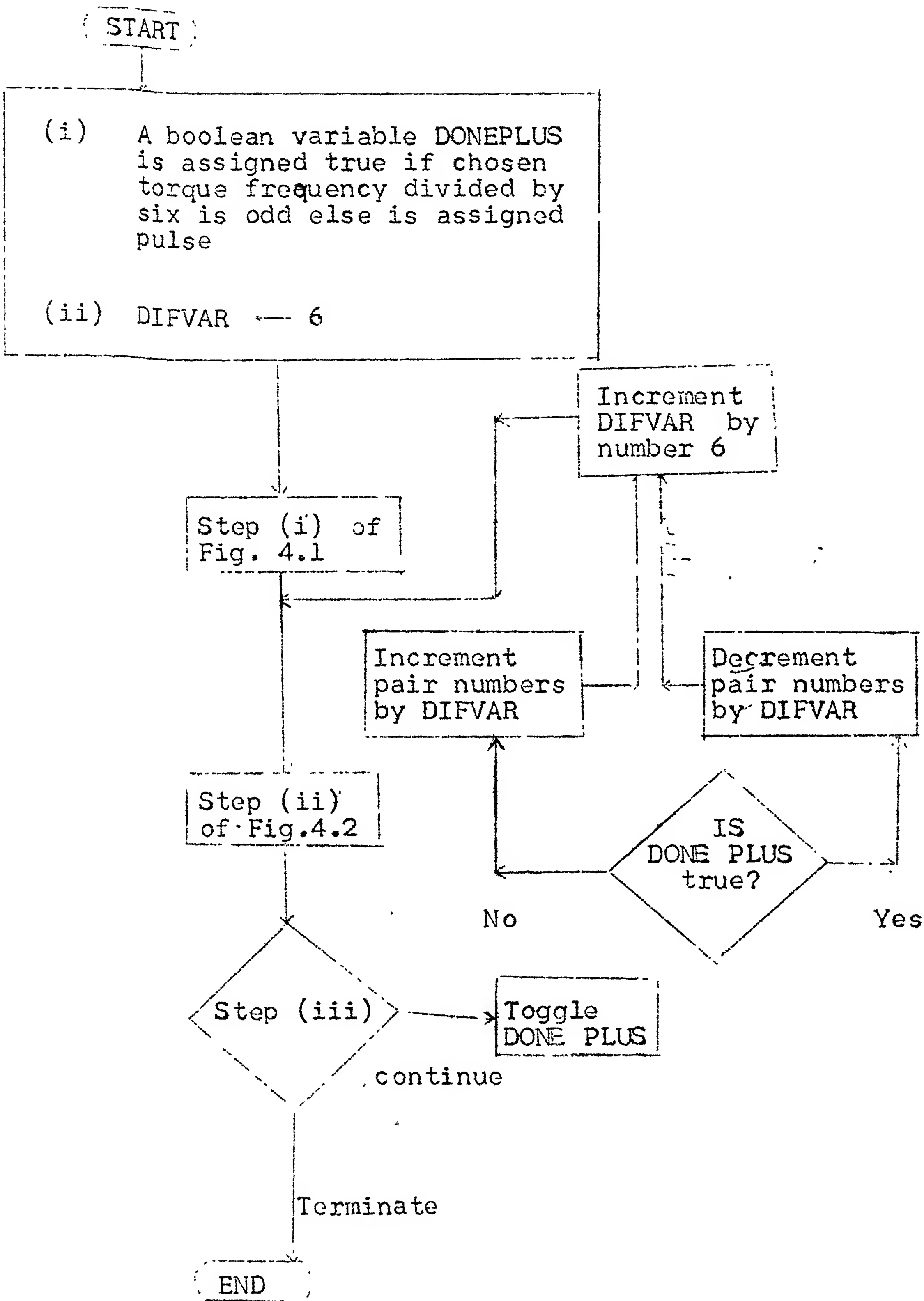


Fig. 4.3 Flow chart for the computation of a particular torque harmonic

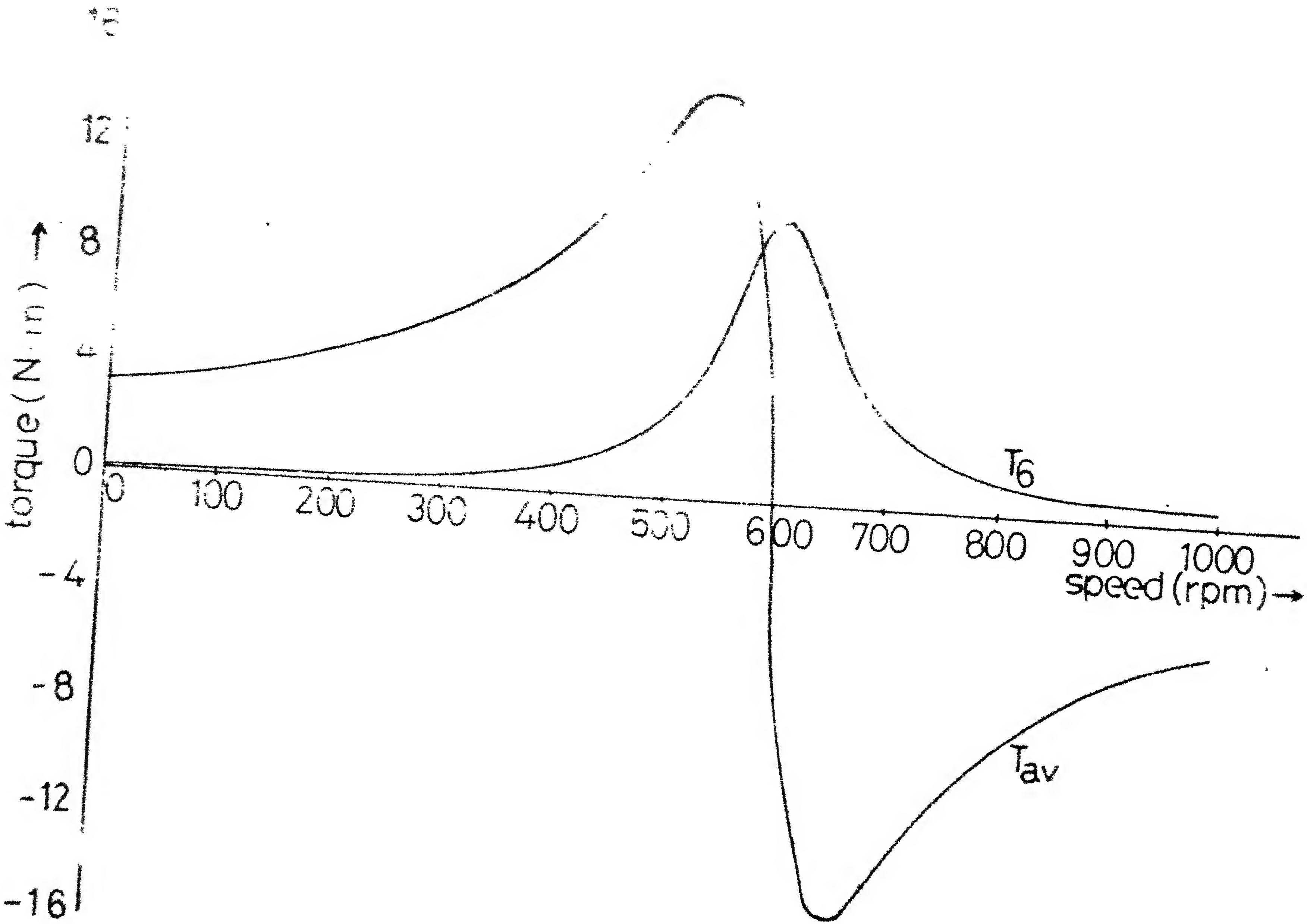


Fig 4.4: Speed Vs Average & Sixth Harmonic Torque

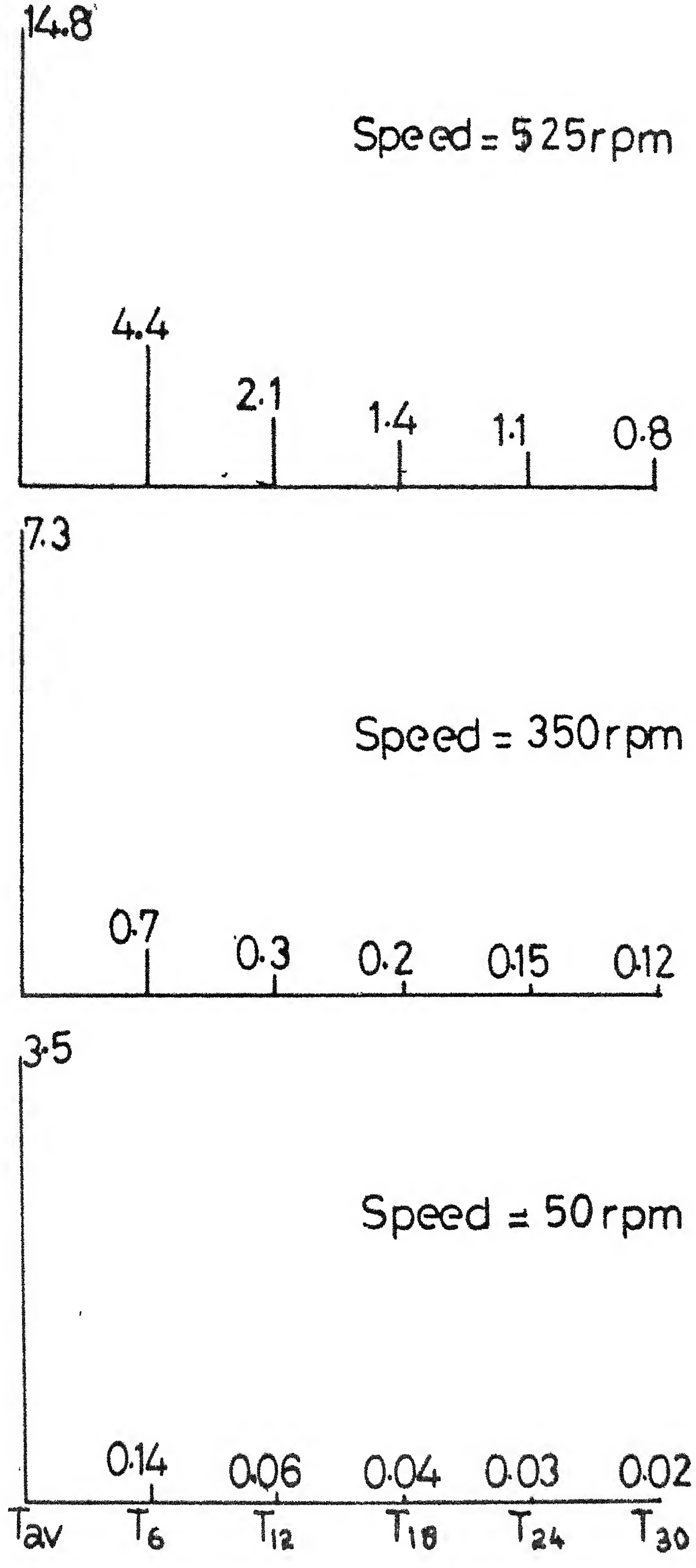


Fig.4.5: Harmonic Torque Spectrum

For current of fig 2.1

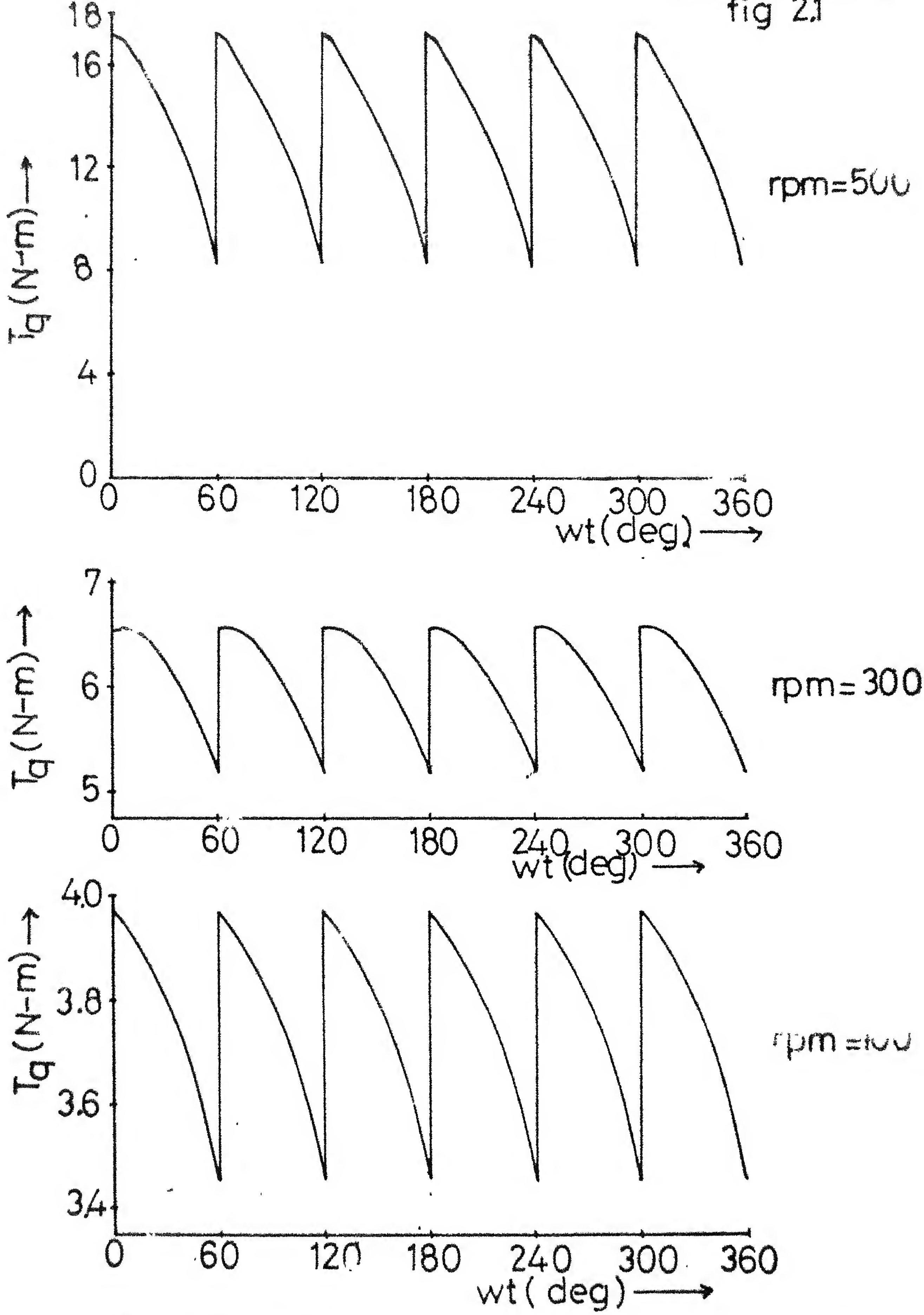


Fig 4.6 : Electromagnetic torque waveform

Table 4.1

Torque Harmonic Expressions

S.No.	Type of the current harmonic	Final torque expression	Comments
1.	Both n and m of the type (3p+1)	$2I_sI_rM_c \sin[(n-m)\omega t-(\alpha_s-\alpha_r)]$	(n-m) is of the type '3p'
2.	Both 'n' and of the type(3p+2)	$-2I_sI_rM_c \sin[(n-m)\omega t-(\alpha_s-\alpha_r)]$	(n-m) is of the type '3p'
3.	n of type (3p+1) and 'm' of type (3p+2)	$2I_sI_rM_c \sin[(n+m)\omega t-(\alpha_s+\alpha_r)]$	(n+m) is of the type '3p'
4.	'n' of type (3p+2) and 'm' of type (3p+1)	$-2I_sI_rM_c \sin[(n+m)\omega t-(\alpha_s+\alpha_r)]$	(n+m) is of the type '3p'
5.	either n or m is of the type 3p	0	when neither of (n-m) and (n+m) is triplen or both n and m are triplen

TABLE 4.2
****

TORQUE HARMONIC VALUES VIA DIFFERENT METHODS

FOR IDC=6.0 amps. , ROTOR SPEED = 520 RPM ,
INV. FREQ. = 20 Hz AND Wc = 825 rads/sec

COMPUTATION THROUGH FREQ. DOMAIN TECHNIQUE

HARMONIC	6	12	18	24
TORQUE	3.87153	1.57511	0.76771	0.36666

VALUES OBTAINED BY HARMONIC COMPONENTS OF
THE TIME DOMAIN SOLUTION

HARMONIC	6	12	18	24
TORQUE	3.87195	1.57518	0.76770	0.36672

TORQUE VALUE OBTAINED VIA FREQ. DOMAIN BUT CONSIDERING
TWO HARMONIC IN CURRENT ,WHICH HAVE DOMINANT CONTRIBUTION
TO THE TORQUE HARMONIC

HARMONIC	6	12	18	24
TORQUE	3.87069	1.57500	0.76777	0.36686
CURRENT HAR. CONSIDERED	5,7	11,13	17,19	23,25

CHAPTER 5

REDUCTION OF TORQUE HARMONICS BY MODULATION OF D.C. INPUT CURRENT

5.1 INTRODUCTION

The induction motor being fed by a current source inverter produces harmonic torques. The harmonic torques can be reduced by modification of the stator current waveform This can be done either by modulating the dc current input of the inverter or by modulation within the inverter. In this chapter the former technique of modulation has been studied. The latter technique is referred as pulse width modulation in the literature [10].

The aim of the modulation is to reduce the harmonic torques for the same average torque. The interest is directed more toward reducing the dominant torque harmonics, 6th and 12th. The higher order harmonics apart from being lower in magnitude are not reflected in the shaft speed because of the large inertia of the mechanical system.

The reduction in the torque harmonic can be noted by the increase in factor (T_{av}/T_n) after modulation, where 'T_{av}' and 'T_n' refer to average and nth harmonic torques respectively. This is so because torque can be seen to be proportional to the square of stator current, from equation (4.38). Thus

the inverter input current when multiplied by the factor $[(T_n)\text{no mod}/(T_n)\text{mod}]^{1/2}$ gives the average torque after modulation equal to case without modulation. This allows us to compare the relative contents of harmonic torques of two torque spectrum of different average torques.

It has been shown that the modulating waveform has to fulfill a general requirement so as to produce a balanced three phase stator current. This modulating current should repeat after every 60° interval, that is, the modulating current frequency should be six times the inverter frequency.

In this chapter the cases of modulating current waveform of exponential or of consinusoidal nature have been studied. Having assumed a modulating current, satisfying the above requirements, the torque produced by the motor has been computed both in time domain and frequency domain (Sec. 4.3).

Time domain solution is obtained by first evaluating pseudo rotor currents using method in Section 2.2. These expression when substituted in equation (4.39) gives the time domain torque expression.

For computation of the torque spectrum in frequency domain, the harmonic components of the stator dq current is obtained through a computer program. The harmonic components of pseudo rotor currents are evaluated by proceeding as in Section 2.3. Then procedure in Sec. 4.3.2 can be used to compute the torque spectrum.

In Section 5.2 the origin of the general requirement on the modulating waveform has been shown. In Section 5.3 the case of exponential waveform as the modulating waveform has been studied. In this case the amplitude and the time constant of the exponential waveforms are the parameters for the study. In Section 5.4 the case of consinusoidal modulating waveform has been studied. In this case, the magnitude, frequency and the phase of the modulating waveform act as parameters for study.

It should be noted here that in this chapter, for the purpose of simplification of analysis, the current source inverter has been assumed to be ideal.

5.2 GENERAL REQUIREMENTS OF MODULATING WAVEFORM

The current source inverter produces balanced three phase line currents. These currents have following property :

(i) They are displaced from one another by 120°

(ii) Each of the line current is antisymmetric (i.e.,

$$i(\theta) = -i(\pi + \theta)).$$

Let the current during interval I in phase 'a', be a general function of time f(t), where t refers to the time with origin at the start of an interval under consideration. For example, f(t) may be a exponential waveform as shown in Fig. 5.1. Then the intervals III and V of phases 'b' and 'c' respectively will also have the same current f(t) due to property (i) above.

Interval IV phase 'a' current is $-f(t)$ due to property (ii) above. Thus phase 'b' and 'c' have currents of type $-f(t)$ during intervals VI and II respectively.

Thus during all six intervals, the conducting phases carry the same function of current $f(t)$. Therefore, inverter input current should be same during all intervals to satisfy the above condition. This implies that the modulating current should repeat after every 60° interval, that is, it should have a frequency equal to six times the inverter frequency.

5.3 EXPONENTIAL MODULATION

The general expression for the exponentially modulated inverter input d.c. current, $i(t)$, during a 60° interval of inverter can be written as

$$i(t) = I + K(e^{\beta t} - 1) \qquad (5.1)$$

where 'K' and 'β' are the magnitude and inverse of time constant of the modulating waveform. 'I' is the unmodulated input d.c. current. Here the origin of time, t, is at the start of each of the intervals. The three phase currents obtained by this modulated inverter input d.c. current are drawn in Fig. 5.1. To study the effect of the modulation on torque harmonics, both time and frequency domain solutions for the torque are obtained.

From the time domain solution of the torque it has been shown in this section that the torque harmonics can be reduced to zero for the case of stationary rotor, if K and β are chosen to be 'I' and 'a' (eqn. (2.18)) respectively. For the rotating rotor case it has been proven that it is not possible to get a constant average torque by using the exponential modulation.

The effects of variations in K and β on the harmonic torque spectrum have been studied through frequency domain torque computation technique. The results obtain show that any exponential modulation improves the performance index. T_{av}/T_n, for all rotor speeds below the synchronous speed. But in case of speeds greater then synchronous speed, there is a deterioration in this performance index if due to any exponential modulation.

5.3.1 Time domain analysis for the exponential modulation

In this section, the analysis for the stationary and rotating rotor cases has been done separately. It has been shown that a constant average torque can be obtained for the case of stationary rotor but not during rotating rotor by exponential modulation.

5.3.1.1 Case of stationary rotor

During interval I, Fig. 5.1, the currents are

$$i_a = K\, e^{\beta t} + I^* \tag{5.2}$$

$$i_b = -(K\, e^{\beta t} + I^*) \tag{5.3}$$

$$i_c = 0 \tag{5.4}$$

where $$I^* = (I - K) \tag{5.5}$$

These can be transformed to dq frame as

$$i_{d1} = [K\, e^{\beta t} + I^*] \tag{5.6}$$

$$i_{q1} = -\frac{1}{\sqrt{3}}\,[K\, e^{\beta t} + I^*] \tag{5.7}$$

The time domain solution for the torque during this interval I can be obtained using equation (4.40). For this $X_1(t)$ and $X_2(t)$ have to be calculated.

For the case of stationary rotor, i.e., $\omega_r = 0$, equation (2.51) gives

$$pX_2 = -aX_2 + \frac{M}{L_{22}}\, ai_{q1}$$

This implies

$$(a+p)\; X_2 = \frac{M}{L_{22}}\, ai_d$$

Substituting for i_{d1} in this equation from equation (5.6), gives

$$(a+p)\; X_2 = \frac{M}{L_{22}}\, a[K\, e^{\beta t} + I^*] \tag{5.8}$$

The solution of this equation (5.8) is

$$X_2(t) = C_1 e^{-at} + A_1 e^{\beta t} + B_1 \tag{5.9}$$

where

$$A_1 = \frac{K_2 K}{a+\beta} \tag{5.10}$$

$$B_1 = \frac{K_2 I^*}{a} \tag{5.11}$$

$$K_2 = \frac{M}{L_{22}} a \tag{5.12}$$

and C_1 is a constant.

Similarly, from equation (2.50) we obtain

$$X_1(t) = C_2 e^{-at} + D_1 e^{\beta t} + E_1 \tag{5.13}$$

where

$$D_1 = -\frac{1}{\sqrt{3}} \cdot \frac{K_2 K}{(a+\beta)} \tag{5.14}$$

$$E_1 = -\frac{K_2}{\sqrt{3}} \frac{I^*}{a} \tag{5.15}$$

and C_2 is a constant.

Substituting for i_{d1}, i_{q1}, X_1 and X_2 from equations (5.6), (5.7), (5.14) and (5.9) into equation (4.40), we obtain the torque expression during interval I, after simplifying as

$$T_q = -M_c(C_2 + \frac{C_1}{\sqrt{3}}) [I^* e^{-at} + K e^{(\beta-a)t}]$$

The time dependence of torque in above equation can be removed if K and β are chosen as

$$K = I \; ; \qquad \beta = a \tag{5.16}$$

For this choice,

$$T_q = -M_c\left(C_2 + \frac{C_1}{\sqrt{3}}\right) I \tag{5.17}$$

that is, a constant torque is obtained.

The values of C_1 and C_2 depend upon the initial conditions and can be computed by the analysis, as shown in Sec. 2.2. Using the method of Section 2.2, the following values can be obtained

$$\begin{bmatrix} C_1 \\ C_2 \end{bmatrix} = \frac{4}{(4x_1^{*2}+3)} \begin{bmatrix} x_1^* & \sqrt{3}/2 \\ -\sqrt{3}/2 & x_1^* \end{bmatrix} \begin{bmatrix} (x_2^*-x_3^*) \\ -\frac{1}{\sqrt{3}}(x_2^*+x_3^*) \end{bmatrix} \tag{5.18}$$

$$x_1^* = \left(\frac{1}{2} - e^{-\pi a/3\omega}\right)$$

$$x_2^* = A_1 e^{\pi\beta/3\omega} + B_1 \tag{5.19}$$

$$x_3^* = A_1 + B_1$$

Table 5.1 lists the expressions of i_{d1}, i_{q1}, X_1, X_2 and T_q with values of K and β from equation (5.16) for intervals II and III. Here C_3, C_4, C_5 and C_6 are constants which depend on the initial conditions and can be calculated as in Section 2.2. In this, t_2 and t_3 are as defined in Section 2.2. The torque is seen to be constant for intervals II and III for K,β choice of equation (5.16). Intervals IV, V and VI are similar to intervals I,II and III respectively because of symmetry of current about $\omega t = \pi$.

Substituting, from equation (5.16), in equation (5.1), gives

$$i(t) = I\ e^{at} \tag{5.20}$$

This equation (5.20) gives the form of the input current during an interval, which will produce a constant torque for the stationary rotor case.

5.3.1.2 Case of rotating rotor

The pseudo rotor currents are related to stator dq currents from equations (2.52) and (2.53) as

$$(p^2+2ap+a^2+\omega_r^2)X_1 = K_1 a i_{q1} + K_1 \omega_r i_{d1} + K_1 p i_{q1} \tag{5.21}$$

$$(p^2+2ap+a^2+\omega_r^2)X_2 = K_1 a i_{d1} - K_1 \omega_r i_{q1} + K_1 p i_{d1} \tag{5.22}$$

Let us compute the torque expression for a period during interval I i.e. for 'ωt' values between 0 and π/3.

Using values of i_{d1} and i_{q1} for this period in (5.21) and (5.22) from equations (5.6) and (5.7), the expression for X_1 and X_2 are X_2 are obtained as

$$X_1 = C_7 e^{-at} \cos(\omega_r t + \phi_1) + A_2 e^{\beta t} + B_2 \tag{5.23}$$

$$X_2 = C_8 e^{-at} \cos(\omega_r t + \phi_2) + A_3 e^{\beta t} + B_3 \tag{5.24}$$

where

$$A_2 = \frac{K q_1}{\beta^2+2a\beta+a^2+\omega_r^2} \tag{5.25}$$

$$q_1 = K_1(\omega_r - \frac{a + \beta}{\sqrt{3}}) \tag{5.26}$$

$$B_2 = \frac{q_2 I^*}{(a^2+\omega_r^2)} \tag{5.27}$$

$$q_2 = K_1(\omega_r - \frac{a}{\sqrt{3}}) \tag{5.28}$$

$$A_3 = \frac{Kq_3}{(\beta^2+2a\beta+a^2+\omega_r^2)} \tag{5.29}$$

$$q_3 = K_1(\frac{\omega_r}{\sqrt{3}} + a + \beta) \tag{5.30}$$

$$B_3 = \frac{q_4 I^*}{(a^2+\omega_r^2)} \tag{5.31}$$

$$q_4 = K_1(\frac{\omega_r}{\sqrt{3}} + a) \tag{5.32}$$

$C_7, C_8, \emptyset_1$ and $\emptyset_2$ in equations (5.23) and (5.24) are constns which depend on the initial conditions and can be computed by the analysis as done in Section 2.2.

The expressions for i_{d1}, i_{q1}, X_1 and X_2 are substituted from equations (5.6), (5.7), (5.23) and (5.24) into equation (4.40) to get the expression for torque. After simplification this equation is given as

$$T_q = -M_c[K\ e^{(\beta-a)t}\ C_9 \cos(\omega_r t+\emptyset_3) + KA_4\ e^{2\beta t} +$$

$$+ (KB_4+A_4I^*)e^{\beta t}+C_9I^*e^{-at} \cos(\omega_r t+\emptyset_3)+B_4I^*] \tag{5.33}$$

where

$$A_4 = \frac{4}{3} K\omega_r \frac{K_1}{(\beta^2+2a\beta+a^2+\omega_r^2)} \tag{5.34}$$

$$B_4 = \frac{4}{3} \frac{K_1\omega_r I^*}{(a^2 + \omega_r^2)} \tag{5.35}$$

and $C_9, \emptyset_3$ are obtained from relation

$$C_9 \cos(\omega_r t+\emptyset_3) = C_7 \cos(\omega_r t+\emptyset_1) + \frac{C_8}{\sqrt{3}} \cos(\omega_r t+\emptyset_2) \tag{5.36}$$

That is, C_9 and $\emptyset_3$ depend upon the initial conditions.

To remove the time dependence of equation (5.33) we have the choice of two parameters K and β. Since there is a term of type '$e^{2\beta t}$', to make equation (5.33) time independent, it's

coefficient should be made zero by choice of K or β. This is possible only either K or A_4 is zero. Since we are dealing with rotating rotor case, $\omega_r \neq 0$. For exponential modulation $K \neq 0$. Thus $(K.A_4)$, the coefficient of $e^{2\beta t}$ cannot be reduced to zero.

Thus it is not possible to get an exactly constant torque for the rotating rotor case, using the exponentially modulated waveform.

Though it is not possible to make equation (5.33) independent of time with K and β as parameters, but it is evident from this equation that certain choices of K and β may give lower time variations in torque, then others. This can be seen in a better way through torque spectrum in frequency domain. This has been studied in detail in next section through the frequency domain analysis of the torque spectrum due to the current waveform of Figure 5.1.

5.3.2 Frequency domain analysis for the exponential modulation

The effects on the torque spectrum due to the variations in the exponentially modulated waveform, Fig. 5.1, have been studied in this section. The torque spectrum produced by this stator current waveform has been computed using the method of Sec. 4.3.2.

It is seen from equation (5.1), that for exponential modulating waveform there are two control parameters, β and K.

In this section the effects on torque spectrum due to the variation in the values of β and K of the modulating waveform has been studied for various values of rotor speeds. When the effects due to variations in 'β' are studied, 'K' is assigned a value equal to 'I', which gives no torque harmonics for stationary rotor case, (Sec. 5.3.1.1). Similarly, when effects on torque because of variations in 'K' is studied, 'β' is assigned the value 'a', as this value of 'β' removes torque harmonics for stationary rotor case, Sec. 5.3.1.1.

5.3.2.1 Analysis with the variations in the time constant of the modulating exponential waveform

To study the effect on the torque spectrum due to variation of β, the inverse of modulating waveform time constant, the amplitude of this waveform, K, is taken as I. It is noted that this value of K and β equal to 'a' give a constant torque for stationary rotor case. For this choice of K, the expression for the modulated inverter input current is given by, from equation (5.1) as

$$i(t) = I\,e^{\beta t} \tag{5.21}$$

For the current the torque spectrumccan be obtained for various values of β, at various rotor speeds. Suppose we are interested in reduction of the 6th harmonic of the torque. Fig. 5.2 plots the performance index (T_{av}/T_6) as function of the rotor speed. T_6 refers to 6th harmonic torque and T_{av} to the

average torque. In Fig. 5.2, β, the inverse of the time constant of the modulating waveform has been taken as parameter. The case $\beta = o$, in Fig. 5.2 corresponds to the no modulation case. It is evident from this figure that

(i) For the rotor speeds less then the synchronous speed of the motor there is improvement in the performance with any value of 'β' of the modulating waveform as compared to the no modulation case. As against this, for the rotor speeds greater than the synchronous speed of the motor there is no improvement with the exponential modulation, rather there is a deterioration in the performance.

(ii) It is seen from Fig. 5.2 that there is a optimum value of β, at particular value of rotor speed which gives the best performance. For rotor speeds less then synchronous speeds, this optimum value of β increases as the rotor speed increases.

(iii) (a) The performance index (T_{av}/T_6) is a sharp function of β at rotor speeds much less, then synchronous speed. However, at higher speeds, that is, closer to synchronous speed (T_{av}/T_6) is not very sensitive to values of β. For example : at rotor speed of 300 rpm, (T_{av}/T_6) for $\beta = a$ is 20.3 and for $(\beta=2a)$ is 31.4. Similarly at 200 rpm when β changes from a to O, (T_{av}/T_6) reduces from 36.0 to 17.4. At higher speed say at 560 rpm, (T_{av}/T_6) for $\beta = 12a$ and 16a is 5.87 and 5.06 respectively. At 580 rpm, (T_{av}/T_6) changes from 1.41 to 1.98 for change of β from 12a to 16a.

(b) In view of above, it is possible to keep β constant in the normal operation region of induction motor, i.e. between approximately 90% to 100% of the synchronous speed and still obtain near optimum value of (T_{av}/T_6).

(iv) As the rotor speed tends to zero the optimum value of 'β' approaches the value 'a', as obtained in Sec. 5.3.1.

5.3.2.2 Analysis with the variations in the amplitude of the modulating exponential waveform

To study the effects on torque spectrum due to the variations in the amplitude of exponential modulation, K, the value of β is kept constant. This constant value is taken as 'a', as this choice of β gives constant torque for stationary rotor (Sec. 5.3.1.1). In this section, the interest has been focussec on the reduction of the sixth harmonic torque with variation in K.

Fig. 5.3 plots the performance index (T_{av}/T_6) as the function of rotor speed, varying from zero to synchronous speeds. The case K = 0 in the figure corresponds to the case with no modulation. It is evident from this figure that the effects on torque spectrum due to variations in K are similar to the variations in torque spectrum with β as a parameter. That is, for every value of rotor speed, there is an optimum choice of the value of K, which gives us the maximum performance index, (T_{av}/T_6). Similarly, this index is a strong

function of 'K' at rotor speeds much less then synchronous speeds. However at higher speeds, that is, speeds closes to the synchronous speeds (T_{av}/T_6) is not a very sensitive to the values of K. In view of this, it is possible to keep K constant in the normal operation region of the induction motor and still obtain a near optimum value of (T_{av}/T_6).

It can be seen from Fig. 5.2 that as the rotor speed increases, the optimum value of K for the best performance increases. For rotor speed approaching zero, this optimum value of K approaches the value I, inverter input dc current wit no modulation. This is in fact the optimum value of K, which is obtained for the case of stationary rotor (Sec. 5.3.1.1).

5.3.2.3 Optimum pair of (K,β) for a given speed

It has been observed in Sec. 5.3.2.1 that for a given rotor speed and a assumed constant value of K, there is a value of β, which gives the maximum performance index T_{av}/T_6. Similarly, Sec. 5.3.2.2 shows that when β is taken as constant, this optimum property is observed with respect to the value of K as well. It has been shown in this section that there is a optimum choice of the pair (K,β) which gives a maximum performance index at a speed.

To observe this, the maximum T_{av}/T_6 value is obtained for various choices of β by varying K. The plots of $(T_{av}/T_6)_{max}$ as a function of β and (K) optimum as a function of β has been

plotted in Fig. 5.2 for the case of rotor speed = 500 rpm. From these plots it can be observed that,

(i) There is an optimum value of the pair (K,β) which gives the largest $(T_{av}/T_6)_{max}$ value. For this pair, the $(T_{av}/T_6)_{max}$ is very large and it can be assumed that a near constant torque is obtained with this choice of (K,β) pair.

(ii) As the value of β increases, the value of K which gives the maximum value of (T_{av}/T_6) for the chosen β, goes on decreasing.

Fig. 5.3 plots the optimum (K,β) values as the function of speed. From this plot it is seen that as the rotor speed increase the optimum value of β goes on increasing. It is equal to 'a', at zero rotor speed.

5.4 COSINUSOIDAL MODULATION

The general expression for the cosinusoidally modulated inverter input dc current, i(t), during a 60° interval of inverter can be written as

$$i(t) = I + Am\cos(\omega_m t + \alpha_m) \tag{5.38}$$

where A_m, ω_m, α_m are the amplitude, frequency and the phase angle of the modulating waveform. 'I' refers to unmodulated d.c. current. The effects on the torque harmonics due to such a modulation has been studied in this section with both the time and frequency domain torque solutions.

From the time domain solution of the torque it has shown that it is not possible to get a constant average torque with a cosinusoidal modulation. The effects of variations in A_m, ω_m and α_m on the harmonic torque spectrum have been studied through the frequency domain torque computation technique.

5.4.1 Time domain analysis for the cosinusoidal modulation

In this section, it has been argued that it is not possible to obtain a constant torque with cosinusoidal modulation. The cases of stationary and rotating rotors are dealt together.

Let us compute the torque expression for a period of interval I i.e. for 'ωt' values given by

$$o < \omega t < \frac{\pi}{3}$$

For this period, the three phase currents for the dc input current given by equation (5.38), are

$$i_a = I+A_m \cos(\omega_m t+\alpha_m) \tag{5.39}$$

$$i_b = -[I+A_m \cos(\omega_m t+\alpha_m)] \tag{5.40}$$

$$i_c = 0 \tag{5.41}$$

These phase currents can be transformed to dq currents from equations (1.9)and(1.10) as

$$i_{d1} = I+A_m \cos(\omega_m t+\alpha_m) \tag{5.42}$$

$$i_{q1} = -\frac{1}{\sqrt{3}} [I+A_m \cos(\omega_m t+\alpha_m)] \tag{5.43}$$

The expressions for X_1 and X_2 can be obtained by solving equations (5.21) and (5.22) with above values of i_{q1} and i_{q2}. This gives

$$X_1(t) = C_{10}\, e^{-at} \cos(\omega_r t+\phi_4)+D_1 \cos(\omega_m t+\alpha_m)+D_2 \sin(\omega_m t+\alpha_m)+E_1 \tag{5.44}$$

$$X_2(t) = C_{11}\, e^{-at} \cos(\omega_r t+\phi_5)+D_3 \cos(\omega_m t+\alpha_m)+D_4 \sin(\omega_m t+\alpha_m)+E_2 \tag{5.45}$$

where

$$D_1 = Am\,[q_2(a^2+\omega_r^2+\omega_m^2)-2aq\omega_m]/E_3 \tag{5.46}$$

$$E_3 = -[4a^2\omega_m^2+(a^2+\omega_r^2-\omega_m^2)^2] \tag{5.47}$$

$$q_2 = K_1(\omega_r - \frac{a}{\sqrt{3}}) \tag{5.48}$$

$$q = \frac{K_1\ \omega_m}{\sqrt{3}} \tag{5.49}$$

$$D_2 = Am[2aq_2\omega_m+q(a^2+\omega_r^2-\omega_m^2)]/E_3 \tag{5.50}$$

$$E_1 = \frac{q_2\ I}{a^2+\omega_r^2} \tag{5.51}$$

$$D_3 = Am[q_4(a^2+\omega_r^2-\omega_m^2)-2aq_5\omega_m]/E_3 \tag{5.52}$$

$$q_4 = K_1(a+ \frac{\omega_r}{\sqrt{3}}) \tag{5.53}$$

$$q_5 = -K_1\ \omega_m \tag{5.54}$$

$$D_4 = Am[2aq_4\omega_m+q_5(a^2+\omega_r^2-\omega_m^2)]/E_3 \quad (5.55)$$

$$E_2 = \frac{q_4 \ I}{a^2+\omega_r^2} \quad (5.56)$$

C_{10} and C_{11} are the constants which depend on the initial conditions. These can be evaluated by the analysis as done in Sec. 2.2.

The expressions for i_{d1}, i_{q1}, X_1 and X_2 obtained are substituted in equation (4.40) to get the expression for torque. After simplification this equation is given as

$$\begin{aligned} T_q = & -M_c[C_{12}\ I\ e^{-at}\cos(\omega_r t+\phi_6)+ID_5\cos(\omega_m t+\alpha)+ID_6\sin(\omega_m t+\alpha) + \\ & + C_{12}Me^{-at}\cos(\omega_r t+\phi_6)\cos(\omega_m t+\alpha) +(\tfrac{1}{2})M\ D_5\cos(2\omega_m t+2\alpha) + \\ & +(\tfrac{1}{2})M\ D_6\sin(2\omega_m t + 2\alpha) + (\tfrac{1}{2})M\ (D_5+D_6)] \end{aligned} \quad (5.57)$$

where

$$D_5 = 4M\ K_1\ \omega_r(a^2+\omega_r^2-\omega_m^2)/(3E_1) \quad (5.58)$$

$$D_6 = 8M\ a\ K_1\ \omega_r\ \omega_m/(3E_1) \quad (5.59)$$

C_{12} and ϕ_6 are obtained from equation

$$C_{12}\cos(\omega_r t+\phi_6) = C_{10}\cos(\omega_r t+\phi_4)+ C_{11}\cos(\omega_r t+\phi_5) \quad (5.60)$$

If equation (5.57) is to give a constant value, we should satisfy following relations

$$I[D_5 \cos(\omega_m t+\alpha) + D_6 \sin(\omega_m t+\alpha)] = 0 \quad (5.61)$$

$$M[D_5 \cos(2\omega_m t+2\alpha) + D_6 \sin(2\omega_m t+2\alpha)] = 0 \quad (5.62)$$

$$MC_{12} = 0 \quad (5.63)$$

$$IC_{12} = 0 \quad (5.64)$$

equation (5.62) can be written as

$$M\sqrt{(D_5^2+D_6^2)}\cos(2\omega_m t+2\alpha+\phi_7) = 0 \quad (5.65)$$

where

$$\tan\phi_7 = -(D_6/D_5) \quad (5.67)$$

Equation (5.65) is satisfied only if $M = 0$ or D_5 and D_6 are zero (5.68)

$M = 0$ implies the case of no modulation and is not of interest. Thus, we should have

$$D_5 = 0 \quad (5.70)$$

$$D_6 = 0 \quad (5.71)$$

and from equation (5.63)

$$C_{12} = 0 \quad (5.72)$$

Equations (5.70) ato (5.72) are the only possible condition which satisfy equation (5.61) to (5.64), for cosinusoidal modulation. Substituting these in equation (5.57) gives

$$T_q = 0 \tag{5.73}$$

This corresponds to the trivial, case of no stator current. Thus it is not possible to reduce current harmonics to zero, using the cosinusoidal modulation.

5.4.2 Frequency domain analysis for the cosinusoidal modulation

The effects on the torque spectrum due to the variations in the amplitude, frequency and the phase of the cosinusoidal modulating waveform have been studied in this section. The torque spectrum produced by the assumed stator current is computed using the method of Section 4.3.2.

The torque spectrum for the cases with ω_m, of equation (5.38), equal to 3,6,12 and 18 times the inverter frequency have been studied. When studying the nature of torque spectrum it is found that for a particular choice ω_m and A_m there is a value of α_m which gives the maximum performance index (T_{av}/T_6). Also there is a optimum combination of amplitude and a frequency which gives the lowest torque harmonics.

The results have been summarised as follows :

(i) $\underline{\omega_m = 6\omega}$: Table 5.2 gives the 6th and the average torque for various amplitudes and phases for the case of ω_m equal to 6ω and a fixed constant rotor speed. This shows that

(a) There is a distinct phase angle for a every choice of amplitude which gives the maximum performance index for 6th harmonic.

(b) There is a optimum amplitude which for a particular phase angle gives zero sixth harmonic torque.

(c) As the amplitude of modulating waveform increases the phase angle at which the minimum 6th harmonic torque is obtained decreases.

(d) The variations in the average torque produced due to modulation are not much significant.

(ii) In Table 5.3, the effects on 12th harmonic torque for the case of $\omega = 6\omega$ has been taken up. It can be noted that we are able to obtain the reduction in the 12th harmonic torque with modulating current frequency equal to six times the inverter frequency. In this case also the property of the optimum phase and amplitude has been observed.

(iii) $\underline{\omega_m = 3\omega}$: Table 5.4 lists the results for ω_m equal to three times the inverter frequency. This type of harmonic can be present in the inverter input dc current due to imperfect filtering. It is noted that there is an improvement in the ratio of average to 6th harmonic torque and the feature of

optimum amplitude and phase angle is present and is similar case (i). But for this case the change in the average torqu is found to be significant. Even for a particular amplitud of the modulating waveform, there is a significant variation in the average torque with the variations in the phase angle

(iv) $\underline{\omega_m = 12\omega}$: The reduction in 6th harmonic torque by modulating waveform with ω_m equal to twelve times the invert frequency has also been studied. Table 5.5 lists the result obtained for this analysis. It is observed that in this case the variation in the sixth harmonic torque due to the variations in the amplitude and the phase of the modulating wavefo is small, as compared to the frequency of 6ω. Also, here a large scale modulation is required for better performance. He no optimum property with respect to the amplitude of modulati current is observed in the possible range of amplitude of modu lation.

(v) Table 5.6, lists the result for the reduction of 6th harmonic torque with $\underline{\omega_m = 18\omega}$. Here again the variations in 6th harmonic torques are observed to be small. These are even smaller then for the case (iv) above. Here also no optimum property is observed.

Thus it can be said that in case the cosinusoidal modulation is being used, the modulating waveform of the frequency ω_m six times the inverter frequency gives the best control for obtaining the desired optimum performance.

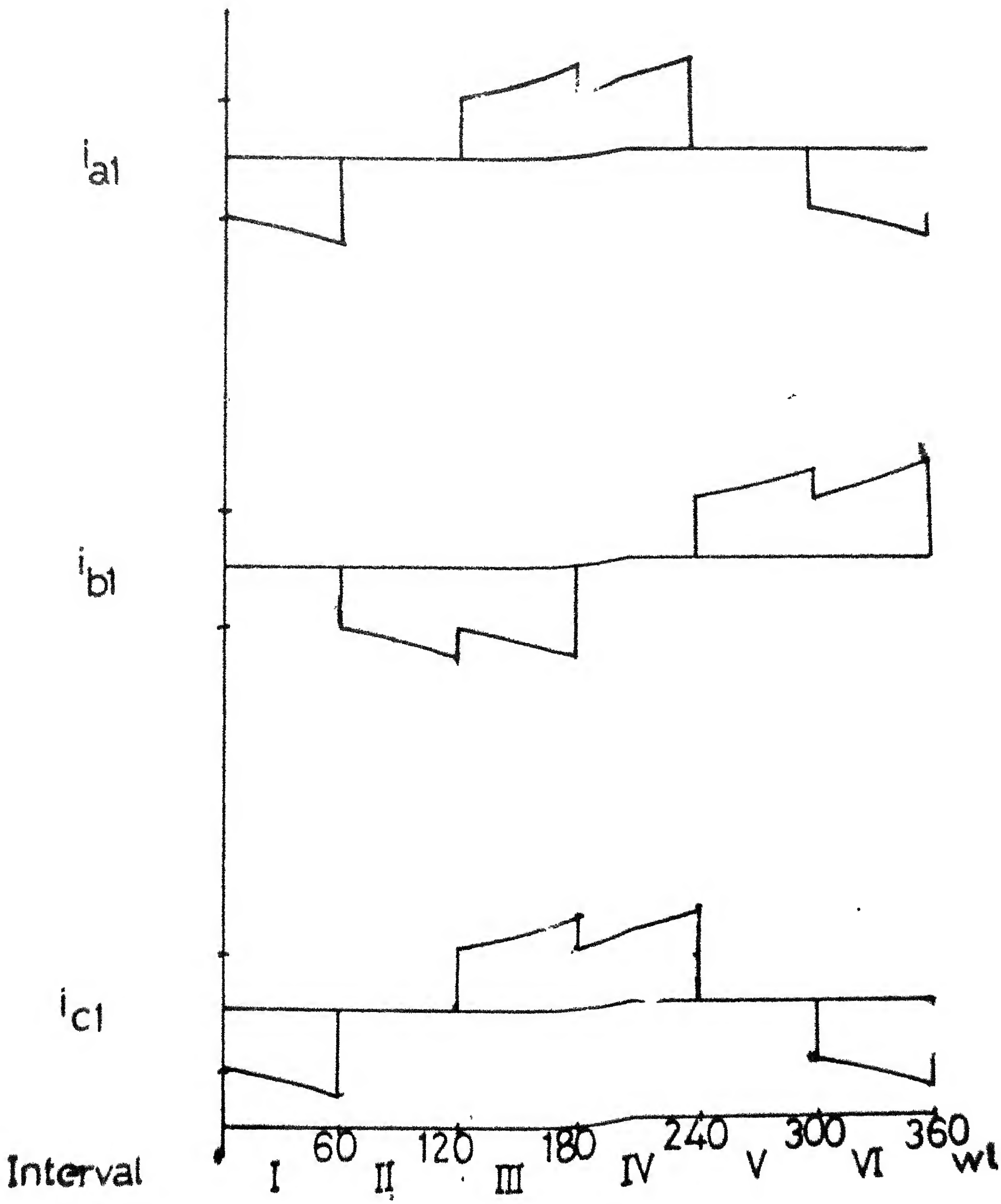


Fig 5.1: Three Phase Current For Exponential Modulation

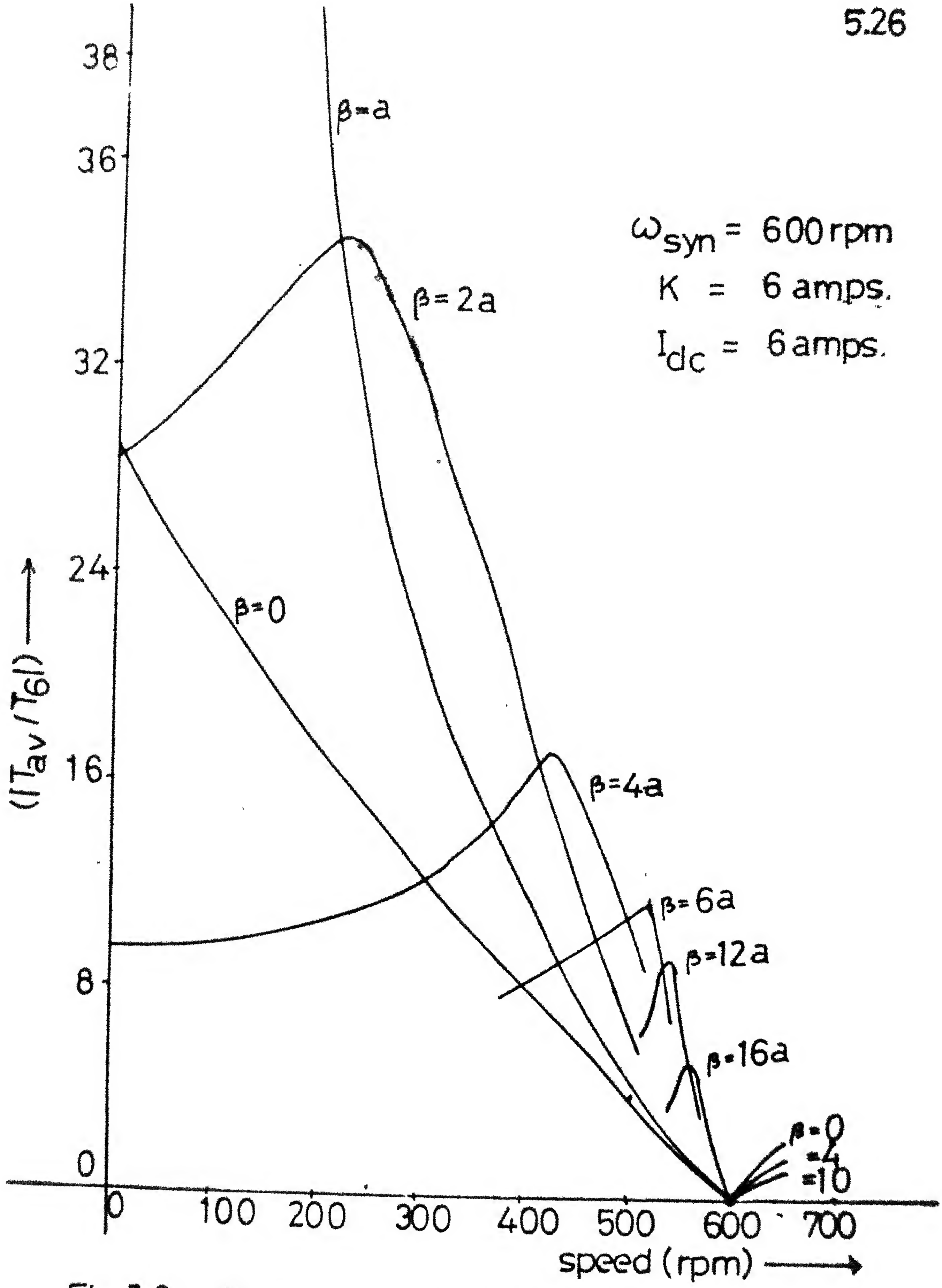


Fig 5.2 : Performance index Vs. speed with $K = I_{dc}$ and variable β.

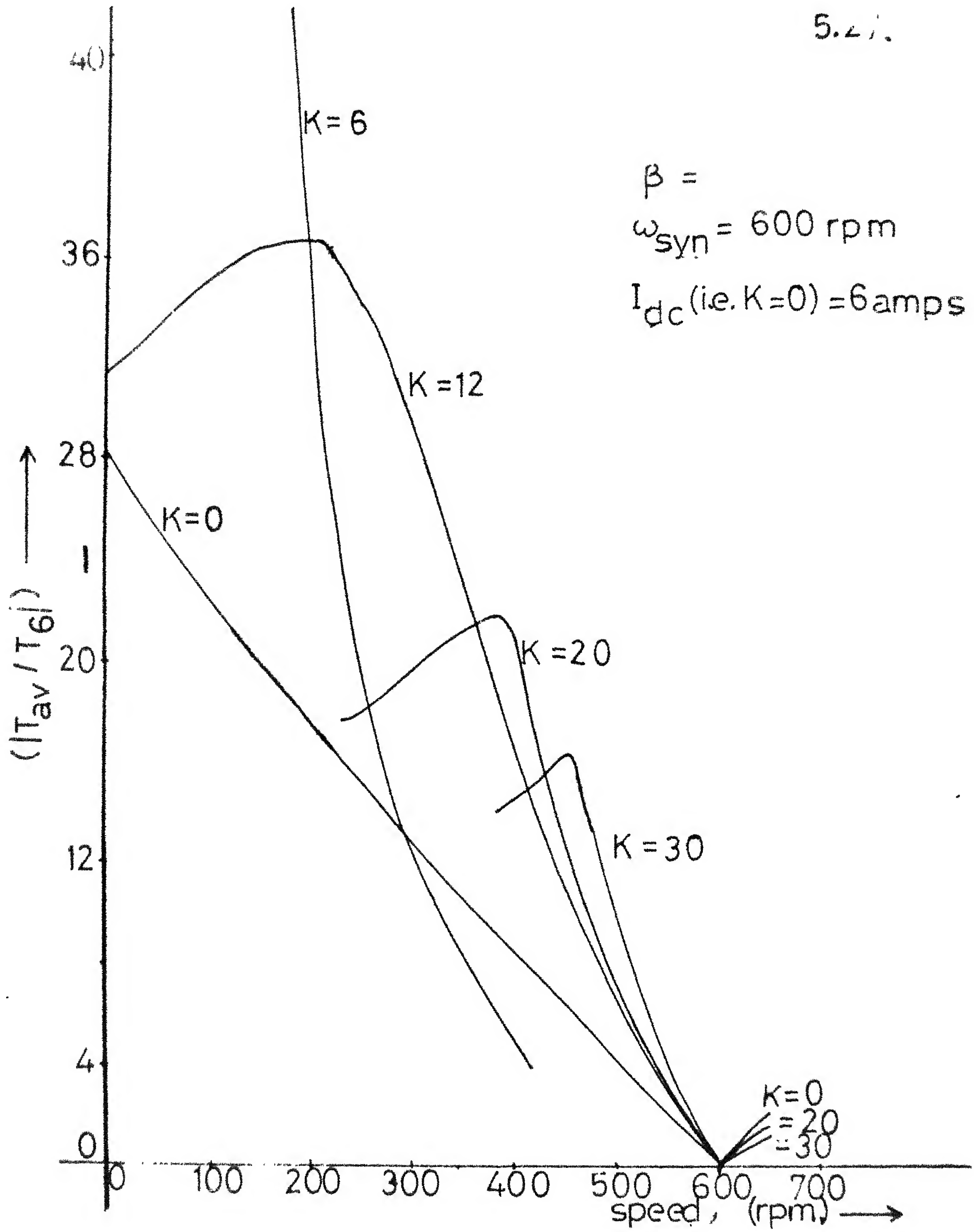


Fig 5.3: Performance index Vs. rotor speed with $\beta = a$ and variable K

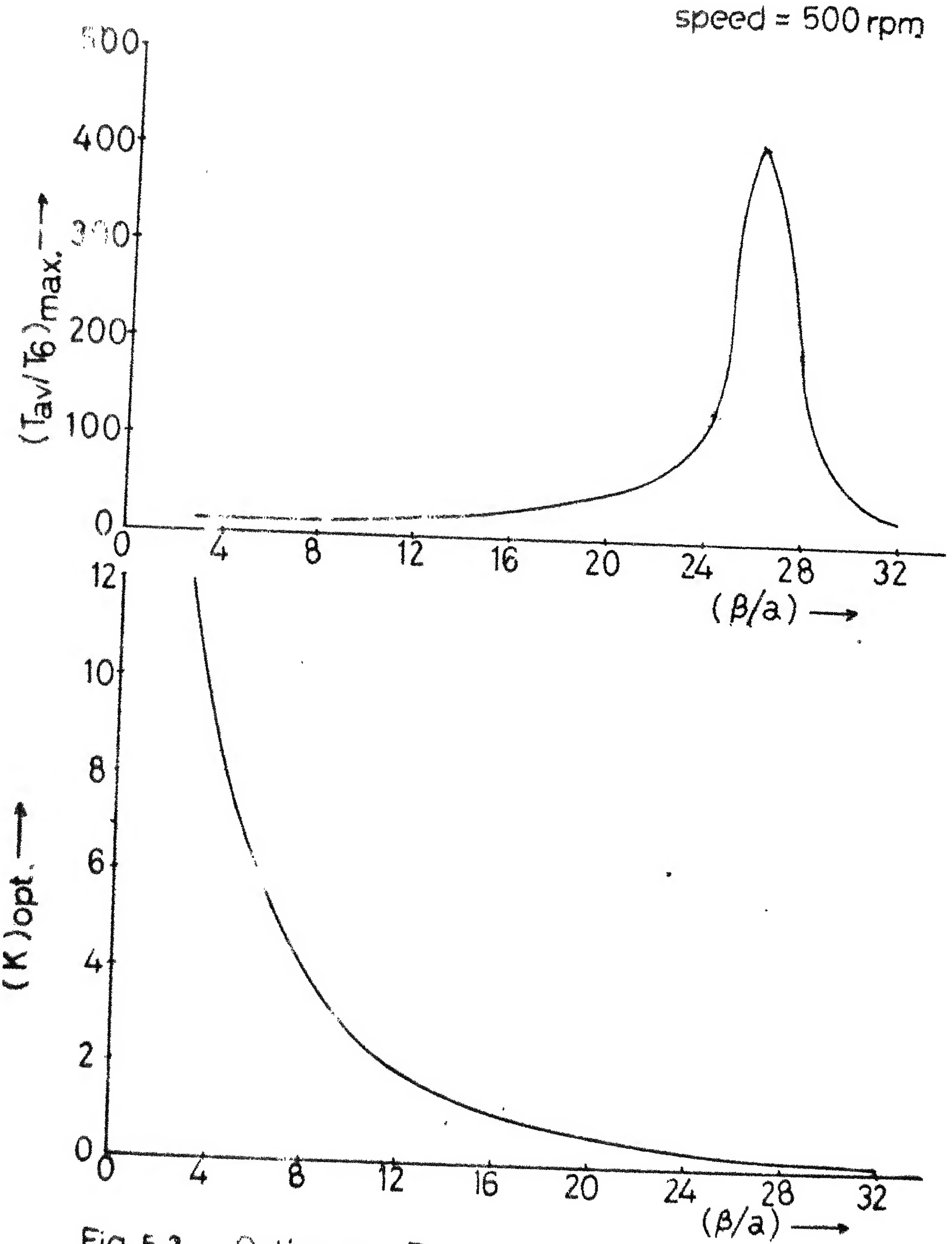


Fig 5.2 : Optimum Exponential Waveform Parameters

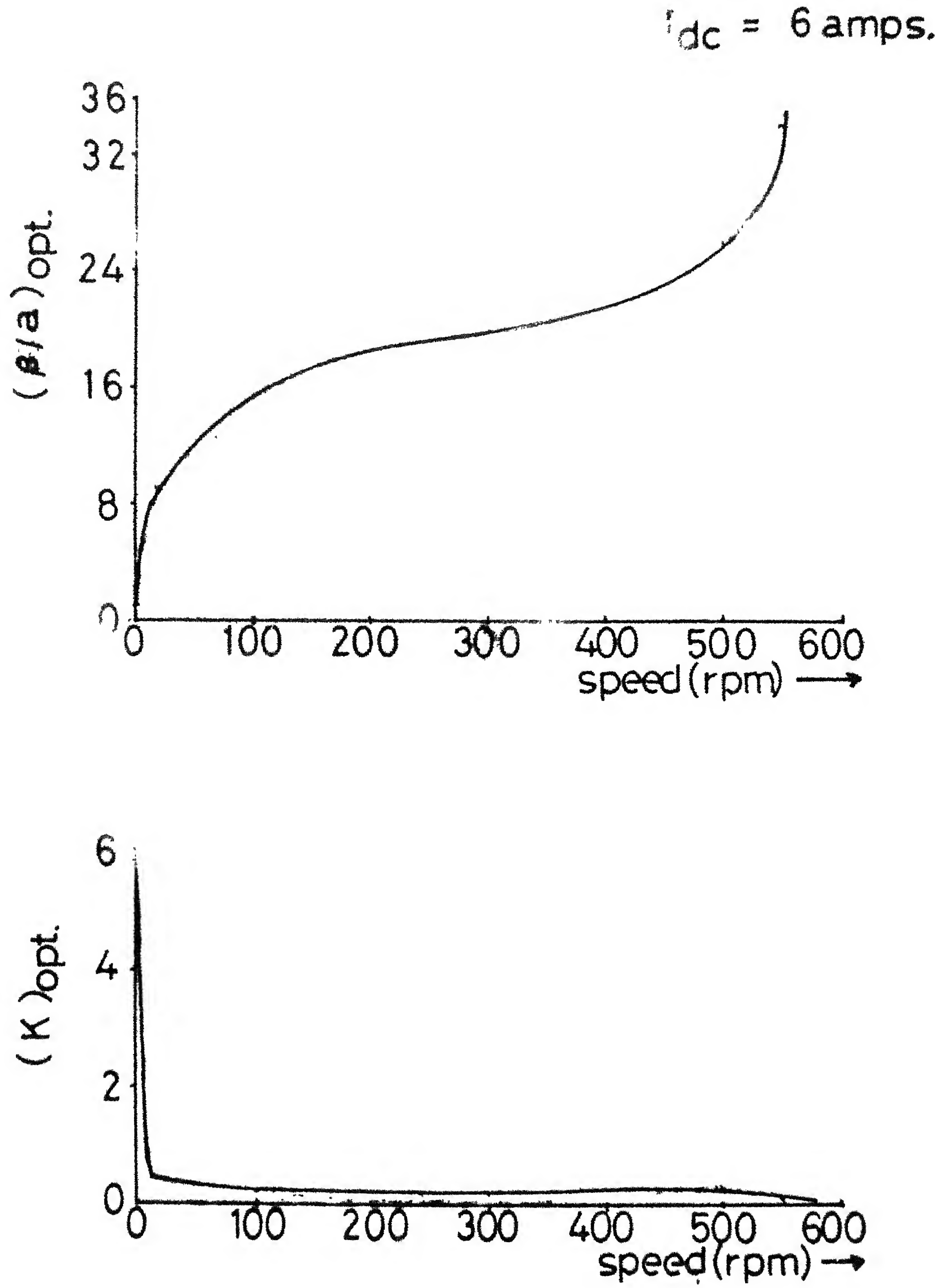


Fig 5.4 : Optimum (K, β) Pair Of Exponentia
Waveform Vs Speed

Table 5.1

Expressions of currents during Intervals II and III for exponentially modulated stator current

Variable	Expression of variable during Interval II	Expression of variables during Interval III
i_{d1}	$(Ke^{\beta t_2}+I^*)$	0
i_{q1}	$\frac{1}{\sqrt{3}}(Ke^{\beta t_2}+I^*)$	$\sqrt{\frac{2}{3}}(Ke^{\beta t_3}+I^*)$
X_1	$C_4e^{-at_2}+\frac{A_1}{\sqrt{3}}e^{\beta t_2}+\frac{B_1}{\sqrt{3}}$	$C_6e^{-at_3}+\frac{2}{\sqrt{3}}A_1e^{\beta t_3}+\frac{2B_1}{\sqrt{3}}$
X_2	$C_3e^{-at_2}+A_1e^{\beta t_2}+B_1$	$C_5e^{-at_3}$
T_q with $K=I$ and $\beta=a$	$-M_c(C_4-\frac{C_3}{\sqrt{3}})I$	$\frac{2}{\sqrt{3}}M_cC_5I$

TABLE 5.2
****

FOR REDUCTION OF 6 HAR. WITH MOD. FREQ. AS 6 TIMES INV. FREQ. AT ROTOR SPEED=520.RPM

AMP OF MOD(AMP)	ANGLE OF MOD(DEG)	VERAGE TORQUE	HARMONIC TORQUE	Tav/Thar
0.000	0.000	14.64299	4.08537	3.58425
0.500	0.000	14.53567	4.30447	3.37688
0.500	36.000	14.55908	3.07431	4.73573
0.500	72.000	14.61895	2.21631	6.59607
0.500	108.000	14.68854	2.52967	5.80650
0.500	144.000	14.74123	3.57657	4.12161
0.500	180.000	14.76073	4.64200	3.17982
0.500	216.000	14.74200	5.46017	2.69991
0.500	252.000	14.68987	5.90231	2.48883
0.500	288.000	14.62037	5.87035	2.49054
0.500	324.000	14.56001	5.31704	2.73837
1.000	0.000	14.43877	5.19065	2.78169
1.000	36.000	14.49025	2.90338	4.99082
1.000	72.000	14.61773	0.41514	35.21164
1.000	108.000	14.75703	2.07946	7.09659
1.000	144.000	14.85474	4.13491	3.59252
1.000	180.000	14.88888	5.76392	2.58312
1.000	216.000	14.85610	6.96377	2.13334
1.000	252.000	14.75956	7.66192	1.92635
1.000	288.000	14.62069	7.68100	1.90349
1.000	324.000	14.49229	6.85735	2.11340
1.500	0.000	14.35228	6.46640	2.21952
1.500	36.000	14.43652	3.73684	3.86330
1.500	72.000	14.63934	1.77840	8.23175
1.500	108.000	14.84846	3.35017	4.43215
1.500	144.000	14.98351	5.49003	2.72922
1.500	180.000	15.02744	7.19967	2.08724
1.500	216.000	14.98529	8.50947	1.76101
1.500	252.000	14.85210	9.36304	1.58625
1.500	288.000	14.64394	9.46618	1.54697
1.500	324.000	14.43984	8.51164	1.69648
1.055	76.500	14.63818	0.00724	2020.5
1.055	76.600	14.63861	0.00097	15101.5
1.055	76.700	14.63904	0.00725	2018.0
1.055	76.800	14.63946	0.01440	1016.4

TABLE 5.3
****

FOR REDUCTION OF 12 HAR. WITH MOD. FREQ. AS
6 TIMES INV. FREQ. AT ROTOR SPEED=520.RPM
WITH IDC=6.0 AMPS

AMP OF MOD(AMP)	ANGLE OF MOD(DEG)	AVERAGE TORQUE	HARMONIC TORQUE	Tav/Thar
0.000	0.000	14.64299	1.98144	7.4
1.000	0.000	14.43877	2.63643	5.5
1.000	36.000	14.49025	2.48176	5.8
1.000	72.000	14.61773	2.16417	6.8
1.000	108.000	14.75703	1.78471	8.3
1.000	144.000	14.85474	1.45822	10.2
1.000	180.000	14.88888	1.31753	11.3
1.000	216.000	14.85610	1.46165	10.2
1.000	252.000	14.75956	1.83353	8.0
1.000	288.000	14.62069	2.25776	6.5
1.000	324.000	14.49229	2.55982	5.7
2.000	0.000	14.27621	3.28186	4.4
2.000	36.000	14.39788	2.95252	4.9
2.000	72.000	14.68378	2.35866	6.2
2.000	108.000	14.96281	1.67168	9.0
2.000	144.000	15.12755	1.03165	14.7
2.000	180.000	15.17643	0.64828	23.4
2.000	216.000	15.12957	0.95773	15.8
2.000	252.000	14.96746	1.75221	8.5
2.000	288.000	14.69012	2.60522	5.6
2.000	324.000	14.40265	3.17808	4.5
3.000	0.000	14.15532	3.91750	3.6
3.000	36.000	14.36588	3.38989	4.2
3.000	72.000	14.84115	2.54987	5.8
3.000	108.000	15.26033	1.63751	9.3
3.000	144.000	15.46141	0.80943	19.1
3.000	180.000	15.50565	0.14185	109.3
3.000	216.000	15.46341	0.63600	24.3
3.000	252.000	15.26667	1.77166	8.6
3.000	288.000	14.85130	3.01507	4.9
3.000	324.000	14.37406	3.82979	3.8
4.000	0.000	14.07609	4.54330	3.1
4.000	36.000	14.39424	3.79217	3.8
4.000	72.000	15.08983	2.72842	5.5
4.000	108.000	15.64959	1.66758	9.4
4.000	144.000	15.85632	0.90505	17.5
4.000	180.000	15.87653	0.75962	20.9
4.000	216.000	15.85762	0.86196	18.4
4.000	252.000	15.65718	1.91391	8.2
4.000	288.000	15.10421	3.48007	4.3
4.000	324.000	14.40652	4.51192	3.2

TABLE 5.4
****

FOR REDUCTION OF 6 HAR. WITH MOD. FREQ. AS
3 TIMES INV. FREQ. AT ROTOR SPEED=520.RPM
WITH IDC=6.0 AMPS

AMP OF MOD(AMP)	ANGLE OF MOD(DEG)	AVERAGE TORQUE	HARMONIC TORQUE	Tav/Thar
0.000	0.000	14.64299	4.08537	3.6
0.500	0.000	14.67053	5.66336	2.6
0.500	36.000	13.29620	4.93793	2.7
0.500	72.000	12.47106	3.94681	3.2
0.500	108.000	12.46395	2.99474	4.2
0.500	144.000	13.27705	2.42174	5.5
0.500	180.000	14.64580	2.55542	5.7
0.500	216.000	16.07634	3.33727	4.8
0.500	252.000	16.99415	4.41237	3.9
0.500	288.000	17.00232	5.37417	3.2
0.500	324.000	16.09722	5.84672	2.8
1.000	0.000	14.72843	7.21695	2.0
1.000	36.000	12.03599	5.74623	2.1
1.000	72.000	10.47836	3.92273	2.7
1.000	108.000	10.46519	2.15428	4.9
1.000	144.000	11.99939	0.85713	14.0
1.000	180.000	14.67897	1.48910	9.9
1.000	216.000	17.59626	3.01649	5.8
1.000	252.000	19.52454	4.97940	3.9
1.000	288.000	19.54195	6.83709	2.9
1.000	324.000	17.63972	7.70583	2.3
2.000	0.000	14.93532	10.18541	1.5
2.000	36.000	9.77526	7.01585	1.4
2.000	72.000	7.03065	3.93735	1.8
2.000	108.000	7.00855	1.58291	4.4
2.000	144.000	9.70892	1.99219	4.9
2.000	180.000	14.83639	3.71072	4.0
2.000	216.000	20.89581	4.68565	4.5
2.000	252.000	25.12301	6.87766	3.7
2.000	288.000	25.16206	10.24406	2.5
2.000	324.000	20.98959	11.71798	1.8

TABLE 5.5
****

FOR REDUCTION OF 6 HAR. WITH MOD. FREQ. AS 12 TIMES INV. FREQ. AT ROTOR SPEED=520.RPM WITH IDC=6AMPS

AMP OF MOD(AMP)	ANGLE OF MOD(DEG)	AVERAGE TORQUE	HARMONIC TORQUE	Tav/Thar
0.000	0.000	14.64299	4.08537	3.6
0.500	0.000	14.61425	3.92203	3.7
0.500	36.000	14.62058	3.99079	3.7
0.500	72.000	14.63702	4.09640	3.6
0.500	108.000	14.65628	4.19314	3.5
0.500	180.000	14.67650	4.25021	3.5
0.500	252.000	14.65697	4.10386	3.6
0.500	288.000	14.63776	3.99823	3.7
0.500	324.000	14.62106	3.92519	3.7
1.500	0.000	14.57111	3.60045	4.0
1.500	36.000	14.59369	3.65725	3.8
1.500	72.000	14.64911	4.19937	3.5
1.500	108.000	14.70707	4.45898	3.3
1.500	180.000	14.75787	4.58383	3.2
1.500	252.000	14.70898	4.23337	3.5
1.500	288.000	14.65152	3.89725	3.8
1.500	324.000	14.59542	3.61843	4.0
2.500	0.000	14.54709	3.28704	4.4
2.500	36.000	14.59072	3.79980	3.8
2.500	72.000	14.69325	4.39372	3.3
2.500	108.000	14.79014	4.77433	3.1
2.500	180.000	14.85835	4.92205	3.0
2.500	252.000	14.79303	4.48632	3.3
2.500	288.000	14.69754	3.91029	3.8
2.500	324.000	14.59406	3.33770	4.4
4.000	0.000	14.54690	2.83720	5.1
4.000	36.000	14.63111	3.85196	3.8
4.000	72.000	14.81956	4.80953	3.1
4.000	108.000	14.97523	5.30255	2.8
4.000	180.000	15.04492	5.43660	2.8
4.000	252.000	14.97920	5.07838	2.9
4.000	288.000	14.82707	4.16614	3.6
4.000	324.000	14.63753	2.99266	4.9
5.000	0.000	14.57066	2.55568	5.7
5.000	36.000	14.68793	3.96949	3.7
5.000	72.000	14.94383	5.14113	2.9
5.000	108.000	15.13897	5.67284	2.7
5.000	180.000	15.19319	5.78377	2.6
5.000	252.000	15.14338	5.59496	2.7
5.000	324.000	14.69685	2.83598	5.2
6.000	0.000	14.61354	2.29462	6.4
6.000	36.000	14.76867	4.14218	3.6
6.000	72.000	15.10014	5.49804	2.7
6.000	108.000	15.33498	6.04697	2.5
6.000	180.000	15.36057	6.13382	2.5
6.000	252.000	15.33960	6.19258	2.5
6.000	324.000	14.78045	2.75719	5.4

TABLE 5.6
****

FOR REDUCTION OF 6 HAR. WITH MOD. FREQ. AS
18 TIMES INV. FREQ. AT ROTOR SPEED=520.RPM
WITH IDC=6.0 AMPS

AMP OF MOD(AMP)	ANGLE OF MOD(DEG)	AVERAGE TORQUE	HARMONIC TORQUE	Tav/Thar
0.000	0.000	14.64299	4.08537	3.6
0.500	0.000	14.62872	4.02507	3.6
0.500	36.000	14.63185	4.04398	3.6
0.500	72.000	14.64013	4.07950	3.6
0.500	108.000	14.64991	4.11604	3.6
0.500	144.000	14.65743	4.14048	3.5
0.500	180.000	14.66030	4.14600	3.5
0.500	216.000	14.65771	4.13130	3.5
0.500	252.000	14.65039	4.09996	3.6
0.500	288.000	14.64065	4.06181	3.6
0.500	324.000	14.63219	4.03218	3.6
1.500	0.000	14.60927	3.90550	3.7
2.500	0.000	14.60194	3.78751	3.9
3.500	0.000	14.60675	3.67132	4.0
5.000	0.000	14.63670	3.50100	4.2
6.000	0.000	14.67183	3.39059	4.3
6.000	36.000	14.74586	3.91350	3.8
6.000	72.000	14.90824	4.51036	3.3
6.000	108.000	15.02815	4.81282	3.1
6.000	144.000	15.05698	4.84931	3.1
6.000	180.000	15.05079	4.82955	3.1
6.000	216.000	15.05619	4.89362	3.1
6.000	252.000	15.03139	4.83267	3.1
6.000	288.000	14.91709	4.33743	3.4
6.000	324.000	14.75413	3.59974	4.1

CHAPTER 6

CALCULATION OF CURRENT PROFILE FOR SPECIFIED TORQUE WAVEFORM

6.1 INTRODUCTION

In the previous chapters the form of the stator current of the motor has been assumed and then the torque waveform has been obtained. In this chapter, the inverse problem of the computation of the stator current to obtain a assumed torque waveform has been studied. This stator current is assumed to be produced by an ideal three phase current source inverter whose input dc current is being modulated.

A nonlinear second order equation has been derived for an interval of the inverter. The solution of this equation gives the stator current waveform for the assumed interval. The stator current waveform needs to be computed only for one interval as the inverter input current needs to be modulated at six times the inverter frequency (Sec. 5.2). Thus the stator current during complete inverter cycle can be obtained from the solution of stator current during any one interval.

The equation to be solved is of the second order. The solution of this equation requires two boundary conditions. These have been obtained, using the fact that under steady state, the rotor mmf must move by 60° in space in each T/6

interval so that each interval is similar.

In this chapter, the stator current waveform required to obtain a constant torque has been derived. For this case, the nonlinear second order equation reduces to two first order nonlinear equations. The analytical solution of these equations for the stationary rotor case has been obtained. This gives a exponential modulation as obtained in Section 5.3.1. For the rotating rotor case, these equations have been solved through two techniques, namely phase plane analysis and numerical analysis. The numerical solution has been done by a computer program. The solutions for the stator current which give the constant torque have been thus obtained for various values of the rotor speeds.

In this chapter the interval of the inverter for which the analysis has been done is interval I (Fig. 2.1). It should again be noted here that the inverter has been assumed to be ideal for the analysis.

6.2 DERIVATION OF EQUATIONS FOR STATOR CURRENT TO PRODUCE DESIRED TORQUE WAVEFORM

In this section, the equation for stator current waveform during interval I is obtained, which gives an arbitary torque waveform $T_q(t)$. This equation is a second order nonlinear equations. The two boundary value conditions required to solve this equation have also been obtained in this section.

6.2.1 Derivations of nonlinear equation

Let us assume a general form $i_a(t)$ of the 'a' phase current during interval I, which gives the desired torque waveform $T_q(t)$. During interval I, for the ideal inverter

$$i_b = -i_a \tag{6.1}$$

$$i_c = 0 \tag{6.2}$$

Thus from equations ~~(4.3a)~~ (1.9) and (1.10), the dq stator currents can be obtained as

$$i_{d1} = i_a(t) \tag{6.3}$$

$$i_{q1} = -\frac{i_a(t)}{\sqrt{3}} \tag{6.4}$$

Substituting these values of i_{d1} and i_{q1} in equation (4.~~40~~39), we obtain

$$T_q(t) = -M_c i_a(t)\ [X_1 + \frac{X_2}{\sqrt{3}}] \tag{6.5}$$

In equation (6.5), $T_q(t)$ is an assumed function of time and $i_a(t)$ has to be evaluated.

From equations (2.52) and (2.53)

$$(p^2+2ap+a^2+\omega_r^2)X_1 = K_1 a i_{q1} + K_1\omega_r i_{d1} + K_1 p i_{q1} \tag{6.6}$$

$$(p^2+2ap+a^2+\omega_r^2)X_2 = K_1 a i_{d1} - K_1\omega_r i_{q1} + K_1 p i_{d1} \tag{6.7}$$

where ω_r is rotor speed and a, K_1 are given by equation (2.18) and (2.53a) respectively.

From equations (6.6) and (6.7), we have

$$(p^2+2ap+a^2+\omega_r^2)(X_1 + \frac{X_2}{\sqrt{3}}) = K_1 a(i_{q1} + \frac{i_{d1}}{\sqrt{3}}) +$$

$$K_1 p(i_{q1} + \frac{i_{d1}}{\sqrt{3}}) + K_1\omega_r \left[i_{d1} - \frac{i_{q1}}{\sqrt{3}}\right]$$

Substituting for i_{d1} and i_{q1} in above equation from equations (6.3) and (6.4), we have

$$(p^2+2ap+a^2+\omega_r^2)(X_1 + \frac{X_2}{\sqrt{3}}) = \frac{4}{3} K_1\omega_r i_a \tag{6.8}$$

Substituting for $i_a(t)$ in above equation from equation (6.5) gives

$$(p^2+2ap+a^2+\omega_r^2)(X_1 + \frac{X_2}{\sqrt{3}}) = -\frac{4K_1\omega_r}{(3M_c)} \frac{T_q(t)}{(X_1 + \frac{X_2}{\sqrt{3}})} \tag{6.9}$$

This can be written as

$$(p^2+2ap+a^2+\omega_r^2)(y(t)) = -\left(\frac{4K_1\omega_r}{3M_c}\right) \frac{T_q(t)}{y(t)} \tag{6.10}$$

$$\text{where } y(t) = (X_1 + \frac{X_2}{\sqrt{3}}) \tag{6.11}$$

Equation (6.5) using equation (6.11) gives

$$i_a(t) = -T_q(t)/(M_c\, y(t)) \tag{6.12}$$

Equation (6.10) can be solved to obtain $y(t)$. This on substitution in (6.12) gives the form of stator current which gives the desired torque waveform $T_q(t)$. Equation (6.10 being of order two requires two boundary value conditions. These have been obtained in Sec. 6.2.2.

6.2.2 Derivation of the boundary conditions

The values of dq currents at the end of the inverter period are related to the initial values by the equation (2.77) as

$$X_2(T_c) = (1/2)\, X_2(o) - (\sqrt{3}/2)\, X_1(o) \tag{6.12}$$

$$X_1(T_c) = (\sqrt{3}/2) X_2(o) + (1/2)\, X_1(o) \tag{6.13}$$

where T_c is the time of interval I, that is, $(\pi/3\omega)$, ω being the inverter frequency. Thus, $X_1(T_c)$ and $X_2(T_c)$ correspond to the pseudo rotor currents at the end of interval I and $X_1(o)$ and $X_2(o)$ at the start of the interval. These relations are used to compute two boundary value conditions in terms of $y(t)$.

From equations (2.50) and (2.51)

$$\dot{X}_1(o) = \omega_r\, X_2(o) - aX_1(o) + K_1\, i_{q1}(o) \tag{6.14}$$

$$\dot{X}_2(o) = -\omega_r\, X_1(o) - aX_2(o) + K_1\, i_{d1}(o) \tag{6.15}$$

Here

$$\dot{X}_1(o) = \left.\frac{dX_1}{dt}\right|_{t=o} \tag{6.16}$$

$$\dot{X}_2(o) = \left.\frac{dX_1}{dt}\right|_{t=o} \tag{6.17}$$

From equations (6.14) and (6.15) we have

$$(\dot{X}_1(o) + \frac{\dot{X}_2(o)}{\sqrt{3}}) = \omega_r(X_2(o) - \frac{X_1(o)}{\sqrt{3}}) - a(X_1(o) + \frac{X_2(o)}{\sqrt{3}}) +$$
$$+ K_1(i_{q1}(o) + \frac{i_{d1}(o)}{\sqrt{3}})$$

Using definition of y(t) from equation (6.11) and equations (6.3) and (6.4) in above, we obtain

$$\dot{y}(o) = \omega_r(X_2(o) - \frac{X_1(o)}{\sqrt{3}}) - ay(o) \tag{6.18}$$

where

$$\dot{y}(o) = \left.\frac{dy(t)}{dt}\right|_{t=o} \tag{6.19}$$

From equation (6.12) and (6.13)

$$(X_1(T_c) + \frac{X_2(T_c)}{\sqrt{3}}) = \frac{1}{2}(X_1(o) + \frac{X_2(o)}{\sqrt{3}}) + \frac{\sqrt{3}}{2}(X_2(o) - \frac{X_1(o)}{\sqrt{3}})$$

Using the definition of y(t) from equation (6.11) and substituting for $(X_2(o) - \frac{X_1(o)}{\sqrt{3}})$ from equation (6.18) in above we obtain

$$y(T_c) = 1/2(1 + \frac{\sqrt{3}a}{\omega_r})\, y(o) + \frac{\sqrt{3}}{2\omega_r}\,\dot{y}(o) \tag{6.20}$$

where

$$y(T_c) = y(t=T_c) \tag{6.21}$$

Similarly from equations (2.50) and (2.51)

$$\frac{d}{dt}\left(X_1+\frac{X_2}{\sqrt{3}}\right)\Big|_{t=T_c} = \omega_r\left(X_1(T_c) - \frac{X_1(T_c)}{\sqrt{3}}\right) - a\left(X_1(T_c) + \frac{X_2(T_c)}{\sqrt{3}}\right) + K_1\left(i_{q1}(T_c) + \frac{i_{d1}(T_c)}{3}\right)$$

Using equations (6.11), (6.3) and (6.4) in above, we obtain

$$\dot{y}(T_c) = \omega_r\left(X_2(T_c) - \frac{X_1(T_c)}{\sqrt{3}}\right) - a y(T_c) \tag{6.22}$$

where

$$y(T_c) = \frac{d}{dt}y(t)\Big|_{t=T_c} \tag{6.23}$$

From equations (6.2) and (6.13)

$$\left(X_2(T_c) - \frac{X_1(T_c)}{\sqrt{3}}\right) = 1/2\left(X_2(o) - \frac{X_1(o)}{\sqrt{3}}\right) - \frac{\sqrt{3}}{2}\left(X_1(o) + \frac{X_2(o)}{\sqrt{3}}\right)$$

Using equations (6.11) and (6.18) this gives

$$\left(X_2(T_c) - \frac{X_1(T_c)}{\sqrt{3}}\right) = \frac{1}{(2\omega_r)}\left(\dot{y}(o)+ay(o)\right) - \frac{\sqrt{3}}{2}\,y(o) \tag{6.24}$$

Substituting from equation (6.24) into equation (6.22) we obtain

$$\dot{y}(T_c) = \frac{1}{2}\left(1+\frac{\sqrt{3}a}{\omega_r}\right)\dot{y}(o) - \frac{\sqrt{3}}{2}\,\omega_r\left(1+\frac{a^2}{\omega_r^2}\right)y(o) \tag{6.25}$$

The equations (6.20) and (6.25) gives the desired boundar value conditions required to solve equation (6.10). These relate the end point value and slope of variable y(t) to the initial value and slope of this variable.

6.2.3 Derivation of the first order equations relating y and (dy/dt)

Let a new variable z be defined as

$$py = dy/dt = z \tag{6.27}$$

Then this implies that

$$p^2y = \frac{d}{dt}(dy/dt) = dz/dt = (dz/dy)\,(dy/dt)$$

That is,

$$p^2y = z\,\frac{dz}{dy} \tag{6.28}$$

Substituting equations (6.27) and (6.28) into equation (6.9) we get

$$z\,\frac{dz}{dy} + 2az = -\frac{4}{3}\left(\frac{K_1\omega_r T_q(t)}{M_c}\right)\frac{1}{y} - (a^2+\omega_r^2)y \tag{6.29}$$

Thus the second order equation (6.9) can be reduced to two first order equations given by equations (6.27) and (6.28). The boundary value conditions of equations (6.20) and (6.25) become

$$y(T_c) = \frac{1}{2}\left(1+\frac{\sqrt{3}a}{\omega_r}\right)y(o) + \frac{\sqrt{3}}{2\omega_r}\,z(o) \tag{6.30}$$

$$z(T_c) = \frac{1}{2}\left(1+\frac{\sqrt{3}a}{\omega_r}\right)z(o) - \frac{\sqrt{3}}{2}\,\omega_r\left(1+\frac{a^2}{\omega_r^2}\right)y(o) \tag{6.31}$$

6.2.4 General properties of the solution of the derived equations

There are certain properties of the solution $y(t)$ which can be directly noted from the derived differential equations (6.10) and (6.29)

(i) It can be seen from equation (6.10) that if $y(t)$ is a solution for torque $T_q(t)$, then $(F)^{1/2}\ y(t)$ is also a solution for the torque value of $F.T_q(t)$. Therefore, from equation (6.12) the current should be increased by a factor of $(F)^{1/2}$ to increase the torque by a factor F.

(ii) If $z = f(y)$ is a solution of equation (6.29) then from equation (6.29) it can be seen that

$$\frac{dz}{dy}\bigg|_{y_o, z_o} = \frac{dz}{dy}\bigg|_{-y_o, -z_o}$$

This property eliminate the consideration of two quadrants of the phase plane plot [13] because of the symmetry.

(iii) It is clear from equation (6.10) that if $y(t)$ is a solution of this equation then $-y(t)$ is also a solution. $T_q(t)$ is the desired torque waveform and has the frequency of six times the inverter frequency (Sec.4.2). Thus this property imply that if $y(t)$ is a solution in interval I then $-y(t)$ is automatically the solution in interval IV.

(iv) Let us study the behaviour of equation (6.10) at the point when the derivative of $y(t)$ is zero, i.e., $py = 0$. At such a point the equation (6.10) reduces to

$$(p^2+a^2+\omega_r^2)y = -\frac{4}{3}\frac{K_1\omega_r}{M_c}\frac{T_q(t)}{y(t)}$$

This implies that

$$p^2 y = -\frac{4}{3}\frac{K_1\omega_r}{M_c}\frac{T_q(t)}{y(t)} - (a^2+\omega_r^2)\ y(t) \qquad (6.32)$$

During interval I, $i_a(t)$ is always positive.

Thus from equation (6.12), $y(t)$ is negative during this interval. Thus equation (6.32) implies that d^2y/dt^2 is always positive when dy/dt is zero. Thus, for assumed positive torque function $T_q(t)$, if there is a possible solution of stator current, then the form of ~~$i_a(t)$~~ $y(t)$ during interval I is one of the three types of those shown in Fig. 6.1.

6.3 PHASE PLANE ANALYSIS FOR THE CASE OF CONSTANT TORQUE

In this section the procedure to obtain the solution for the stator current which produces a constant torque has been given.

For the case of constant torque, T_{qc} the equation (6.29) becomes

$$\frac{dz}{dy} = -\frac{1}{z}\left[\frac{\alpha}{y} + \beta^2 y\right] -2a \qquad (6.33)$$

where

$$\alpha = \frac{4}{3}\,\frac{K_1\omega_r T_{qc}}{M_c} \tag{6.34a}$$

$$\beta^2 = (a^2+\omega_r^2) \tag{6.34b}$$

The equation (6.63) gives the relationship between y and its derivative z (= dy/dt). The phase plane plot [13] can, therefore, be obtained from equation (6.33).

The slope of the trajectories dz/dy = S say has following properties.

(i) It tends to infinity as z tends to zero or y tends to zero. We note that from equation (6.34a) that $\alpha > o$ as we are dealing with the case of torque production due to rotor speeds less then synchronous speed and hence T_{qc} is positive.

(ii) If z is very large $\frac{dz}{dy} \simeq -2a$ and is a constant independent of y. This is true therefore in all quadrants of y-z plane.

(iii) Zero slope isocline i.e. M = o

$$\frac{dz}{dy} = o \quad \text{if} \quad z = -\frac{1}{2a}\left(\frac{\alpha}{y} + \beta^2 y\right) \tag{6.35}$$

This defines a curve in the y-z plane.

The locus of points in the phase plane wherein the trajectories have the same slope is referred to as isocline

This curve has a maxima or minima when

$$\frac{dz}{dy} = -\frac{1}{2a}\left(-\frac{\alpha}{y^2} + \beta^2\right) = 0$$

This gives

$$y = y_o = \pm \frac{\sqrt{\alpha}}{\beta} \tag{6.36}$$

Corresponding to this y of equation (6.36)

$$z = z_o = -\frac{1}{2a}\left(\frac{\alpha+\beta^2 y^2}{y}\right)\Big|_{y=y_o} = -\frac{\alpha}{a y_o}$$

or

$$z = z_o = \pm \frac{\sqrt{\alpha}}{a}\,\beta \tag{6.37}$$

(iv) For the isoclin with slope S , i.e.

$$dz/dy = S$$

From equation (6.33), the equation of the curve in y-z plane for the isocline with slope S is

$$z = -\frac{1}{(S+2a)}\left(\frac{\alpha}{y} + \beta^2 y\right) \tag{6.38}$$

This curve has maxima or minima at

$$\frac{dz}{dy} = -\frac{1}{(S+2a)}\left(-\frac{\alpha}{y^2} + \beta^2\right) = 0$$

This gives

$$y = y_m = \pm \frac{\sqrt{\alpha}}{\beta} \tag{6.39}$$

This y_m of equation (6.39) is same as y_o of equation (6.36). As equation (6.39) is independent of S, it implies that all the isocline curves have a minima at the same value of y given by equation (6.39). From the equation (6.38) it is also clear that the isocline curves for different values of S is similar, except for a different constant multiple for z.

(v) It can be seen from equation (6.33) that the reversing of the signs of z and y, we get the same equation. Thus, the phase plane plot need to be plotted only for 2 quadrants.

(vi) From equation (6.33)

$$\frac{d^2z}{dy^2} = -\frac{1}{z}\left(-\frac{\alpha}{y^2} + \beta^2\right) + \left(\frac{\alpha}{y} + \beta^2 y\right)\frac{(-1)}{z^2}\frac{dz}{dy} \tag{6.40}$$

For the zero slope isocline, i.e. for dz/dy = 0, at the extreme point (y_o, z_o) from equation (6.36) $y_o = \pm\frac{\sqrt{\alpha}}{\beta}$. Thus at this point (y_o, z_o) from equation (6.71)

$$\frac{d^2z}{dy^2} = 0 \tag{6.41}$$

Thus the point (y_o, z_o) is a point of inflexion because at this point; $\frac{dz}{dy}$ and $\frac{d^2z}{dy^2}$ are zero.

Using the above information and S as a parameter it is possible to draw a set of isoclines throughout the phase plane. These are then used to determine the trajectories graphically. The

method is shown in Fig. 6.2. The lines from point P, have slopes S_1 and S_2 which are indicated by the short line segments crossing each isocline. They are extended until they meet the isocline indicating slope M_2. The point P_2 is midway between the intersections. The process is then continued. The trajectory is plotted as a smooth curve through the points $P_1, P_2, ..$ which are thus determined.

The trajectory satisfying the boundary value relations (6.30) and (6.31) gives us the desired stator current waveform. To find the stator current satisfying equations (6.30) and (6.31) following procedure is adopted.

Let some arbitary value y(o) be assumed for solution. Then from equations (6.30) and (6.31), the equation for end point is given by

$$z(T_c) = \frac{1}{2}\left(1 + \frac{\sqrt{3}a}{\omega_r}\right)\left(y(T_c) - \frac{1}{2}\left(1+\frac{\sqrt{3}a}{\omega_r}\right)y(o)\right)\frac{2\omega_r}{\sqrt{3}} + $$
$$- \frac{\sqrt{3}}{2}\,\omega_r\left(1+\frac{a^2}{\omega_r^2}\right)y(o)$$

i.e.,

$$z(T_c) = M_1\, Y(T_c) + M_2\, y(o) \tag{6.42}$$

where

$$M_1 = \frac{1}{2}\left(1 + \frac{\sqrt{3}a}{\omega_r}\right) \tag{6.43}$$

$$M_2 = \left[-\frac{\omega_r}{2\sqrt{3}}\left(1+\frac{\sqrt{3}a}{\omega_r}\right)^2 - \frac{\sqrt{3}}{2}\,\omega_r\left(1+\frac{a^2}{\omega_r^2}\right)\right] \tag{6.44}$$

Since y(o) is assumed constant, the equation (6.42) is a equation of straight line. The intersection of this straight line with the trajectory passing through the point y(o) and z(o) gives one value of $y(T_c)$. This is compared with the $y(T_c)$ obtained from equation (6.30). In case these are same, the assumed z(o) for y(o) choice gives the solution else a new trajectory on the line y(o) = constant is followed.

The phase plane plot with the trajectories for the case of T_{qc} = 4 Nt m and ω_r = 50 rpm has been plotted in Fig. 6.3. This has been plotted only for two quadrants because quadrants II and III are similar to quadrants IV and I respectively because of property (ii) (Sec. 6.2.4).

To obtain the solution for different rotor speed, a different phase plane plot has to plotted. It can be shown that with the change of rotor speeds, we can change the axis in such a manner that the isocline plots remain invariant. This can be seen as follows.

Equation (6.38) gives the equation of isocline with slope S. This can be written as

$$\left(\frac{z}{\alpha}\right) = -\frac{1}{(S+2a)}\left(\frac{1}{y} + \frac{\beta^2 y}{\alpha}\right) \tag{6.45}$$

where

$$\frac{\beta^2}{\alpha} = \frac{3(a^2+\omega_r^2)\ M_c}{4\ K_1\ \omega_r\ T_{qc}} \tag{6.46}$$

If the rotor speed is changed, the phase plane plot is now assumed for different torque $T_{qc}|_{new}$ given by

$$T_{qc}|_{new} = \left(\frac{\omega_r|_{old}}{\omega_r|_{new}}\right)\left(\frac{a^2+\omega_r|^2_{new}}{a^2+\omega_r|^2_{old}}\right) T_{qc}|_{old} \tag{6.47}$$

with such a choice of T_{qc}, the right hand side of equation remains the same even for different rotor speed but with new value of constant torque. So, if the new z-axis scale is assumed as (z/α), the isocline plot will remain the same. It should be noted that since the scale of z axis is changed, the value of S for each isocline will change but the plot of isoclinnes need not be changed.

The phase plane solution gives the values of $\dot{y}(t)$ as a function of y. The time interval can be evaluated from this information. The time interval, t_1 between $y(o)$ and y_1 is giver by

$$t_1 = \int_{y(o)}^{y_1} (1/z)dy \tag{6.48}$$

Thus from this information of $y(t)$ as a function of time, the stator current waveform can be evaluated.

Since $y(T_c)$ point corresponds to the end of interval, thus from equation (6.48)

$$T_c = \int_{y(o)}^{y(T_c)} (1/z)dy \tag{6.49}$$

A interval corresponds to a 60^{o} interval. Thus inverter frequency, ω, is given as

$$\omega = \pi/3T_c \tag{6.50}$$

Thus,

$$\omega = \frac{\pi}{3\left[\int_{y(o)}^{y(T_c)} (1/z)dy\right]} \tag{6.51}$$

6.4 GENERAL NUMERICAL SOLUTION FOR THE STATOR CURRENT

In this section, the equation (6.10) has been solved to obtain the stator current waveform which produces the torque waveform $T_q(t)$. For the case of stationary rotor, an analytical solution can be obtained for the stator current waveform which will produce the torque $T_q(t)$. For the case of rotating rotor, the stator current has been obtained by the numerical solution of equations (6.29) and (6.27). These equations are the first order equations corresponding to equation (6.10).

6.4.1 Case of stationary rotor

For the case of stationary rotor, ($\omega_r = 0$), the equation (6.10) reduces as

$$(p^2+2ap+a^2)\ y(t) = 0 \tag{6.52}$$

For the solution of equation (6.52) these are two boundary value conditions. These have been obtained by substituting $\omega_r = 0$ in equations (6.18) and (6.22) which gives

$$\dot{y}(o) = -ay(o) \tag{6.53}$$

$$\dot{y}(T_c) = -ay(T_c) \tag{6.54}$$

The solution of equation (6.52) is

$$y(t) = Y_1 e^{-at} + Y_2 t\, e^{-at} \tag{6.55}$$

where Y_1 and Y_2 are constants. These are evaluated using boundary value conditions. Substituting from equations (6.55) into (6.53) we obtain

$$-aY_1 + Y_2 = -aY_1$$

i.e.,

$$Y_2 = 0 \tag{6.56}$$

The equation (6.54) gives the condition ~~simulation~~ similar as (6.56). Thus, from equation (6.12)

$$i_a(t) = -\left(\frac{1}{M_c Y_1}\right) T_q(t)\, e^{at} \tag{6.57}$$

The equation (6.57) gives the form of stator current which produces the torque waveform $T_q(t)$.

In case $T_q(t)$ is assumed to be constant, it can be seen that equation (6.57) reduces to the form of stator phase current

obtained in Sec. 5.3.1.1, which is shown to produce a constant torque.

6.4.2 Case of rotating rotor

The solution of the equation (6.29) satisfying the boundary value conditions given by equations (6.30) and (6.31) cane be obtained through numerical analysis. It should be noted here that here $T_q(t)$ is assumed to be some arbitary desired function.

Let the variable y be y_o at some arbitary time $t = t_o$ with $z = z_o$. Then from equation (6.29),

$$\left.\frac{dz}{dy}\right|_{y=y_o} = -\frac{1}{z_o}\left[\frac{4}{3}\frac{K_1\omega_r T_q(t_o)}{M_c\, y_o} + (a^2+\omega_r^2)y_o\right] - 2a \tag{6.58}$$

To the first degree of approximation, the values of time and variable z at $y = (y_o+\Delta y)$, will be given as

$$z\Big|_{y=(y_o+\Delta y)} = z_o+\left(\frac{dz}{dy}\right)\Big|_{y=y_o}\Delta y \tag{6.59}$$

$$t\Big|_{y=(y_o+\Delta y)} = t_o+\frac{1}{z_o}\,\Delta y \tag{6.60}$$

where Δy is a small change in y from the value of y_o.

The steps for solving equation (6.29) satisfying equations (6.30) and (6.31), for a assumed constant rotor speed are given as follows.

(i) Arbitary values of $y(o)$ and $z(o)$ are assumed for solving equation (6.39)

(ii) $y(T_c)$, and $z(T_c)$, the end point values of the solution of equation (6.29) are calculated from equations (6.30) and (6.31) respectively, for the assumed values of $y(o)$ and $z(o)$.

(iii) The equation (6.29) is solved from $y(o)$ to $y(T_c)$ in steps of Δy, using equations (6.58) and (6.60) and the value of $z(T_c)$ is obtained.

(iv) Values of $z(T_c)$ obtained in steps, (ii) and (iii) above are compared and the difference is calculated.

(v) Steps (ii) to (v) are repeated with a modified value of z_o, keeping y_o as in step (i), till the difference in step (iv) is reduced below a limit.

(vi) Step (iii) compute the time t, when the value of y has has become $y(T_c)$. This time t corresponds to the value T_c. From this, the inverter frequency, ω, can be computed, as from equation (6.50)

$$\omega = \pi/3T_c \tag{6.61}$$

The step (iii) gives the solution of equation (6.29) when the assumed initial values of $y(o)$ and $z(o)$, give the difference of step (iv) less then a limit. From this, the stator current waveform during this interval can be obtained using equation (6.12).

Having obtained the stator current waveform for an interval of inverter, which is inverter I in our case, the stator current for other intervals can be obtained using the properties given in Sec. 5.2.

This stator current will produce the assumed torque waveform $T_q(t)$.

6.5 NUMERICAL SOLUTION FOR THE CASE OF CONSTANT TORQUE

Equation (6.29) has been solved for the case of constant torque. This equation for $T_q(t) = T_{qc}$ becomes

$$z \frac{dz}{dy} + 2az = -\frac{4}{3}\left(\frac{K_1 \omega_r T_{qc}}{M_c}\right)\frac{1}{y} - (a^2+\omega_r^2)y \qquad (6.62)$$

If a new variables are defined as

$$y^* = \frac{1}{i_a} \quad ; \quad z^* = \frac{dy^*}{dt} \qquad (6.63)$$

Then from equation (6.12), equation (6.62) becomes

$$z^* \frac{dz^*}{dy^*} + 2az^* = -\frac{4}{3}\left(\frac{K_1 \omega_r M_c}{T_{qc}}\right)\frac{1}{y^*} - (a^2+\omega_r^2)\, y^* \qquad (6.64)$$

The boundary conditions in terms of y^* and z^* are similar to equations (6.30) and (6.31) with y and z replaced by y^* and z^* respectively.

This equation is solved using steps given in Section 6.4 through a computer program, whose listing is given in the Appendix B at various values of rotor speeds.

Figs. 6.4 to Fig. 6.9 give the plots of initial points of solution and corresponding inverter frequency for various value of the rotor speeds. From these plots it can seen that

(i) There is a lower limit of frequency in the plots. This is so because these plots have been drawn for a particular rotor speed and the constant torque is assumed to be positive. Thus the synchronous speed corresponding to the inverter frequency has always to be more then the rotor speed.

(ii) The plot of inverter frequency with respect to the initial value of the solution of $y(t)$ is parabola. This parabola opens up as the rotor speed decreases. In case of parabolic form, there is a upper limit of frequency.

(iii) There a lower limit on the value of $y(o)$. This is so because protection of a constant value of torque has been assumed. The current is inversely proportional to $y(t)$. The initial slope of $y(t)$ is negative and so the slope for the solution is always negative (Sec. 6.4.2 point (ii)). Thus the lower limit on $y(o)$ corresponds such a initial current which will produce torque value greater then assumed torque value. Thus there is no solution during this region.

(iv) For a particular value of inverter frequency there is only one solution for stator current which will give us the assumed constant torque.

(v) Points of interest of low slip one on the lower branch of the parabola. On this as the initial value y(o) value in the solution increases, z(o) value becomes more negative and the corresponding value of the inverter frequency decreases.

(vi) From the Fig. 6.4 to Fig. 6.9 it is possible to plot the initial values (y(o), z(o)) for the solution for the various ~~of~~ rotor speeds. This has been drawn for the inverter frequenc of 20 Hz in Fig. 6.10, for the constant torque at all rotor speeds. This plot can be used to obtain the stator current waveform to obtain constant torque at various rotor speeds for the inverter operating at certain constant frequency.

Fig. 6.11 compares the solution of nonlinear equation at 500 rpm to produce a constant torque and the optimum exponential waveform for this torque and rotor speed [sec. 5.3]. It is seen that there is a close correspondence. This implies that exponential waveform corresponds closely to the stator current required to produce constant torque.

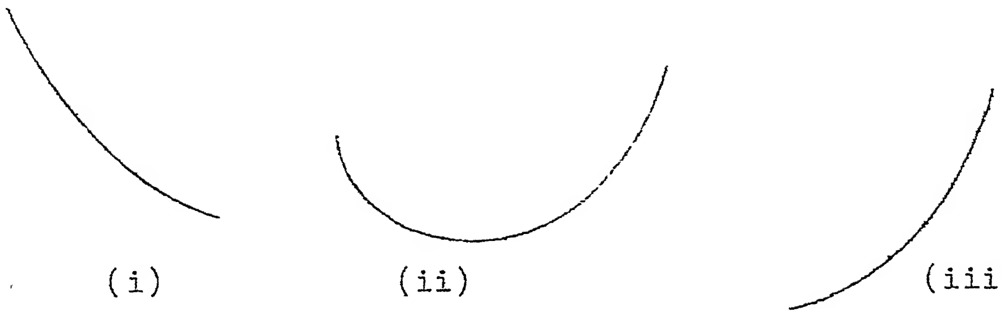


Fig. 6.1 Possible waveshape of ~~a phase current~~ y(t) to produce positive torque function

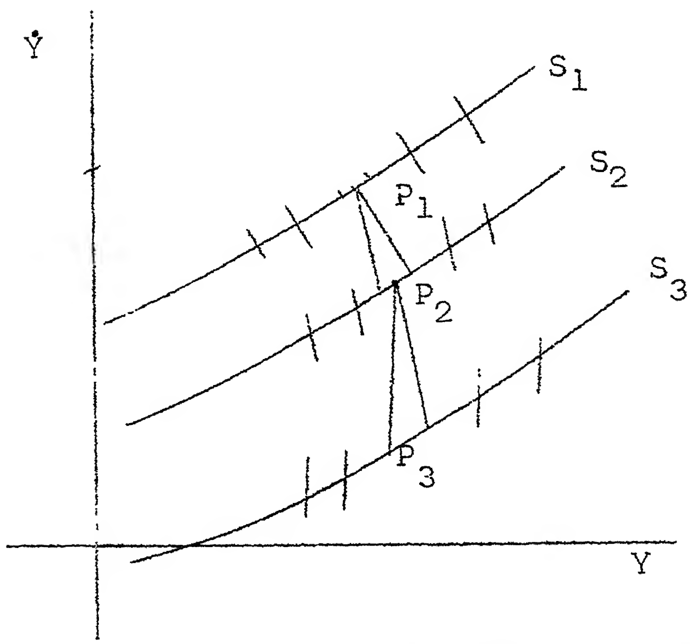


Fig. 6.2 Method for using isoclines to determine trajectories

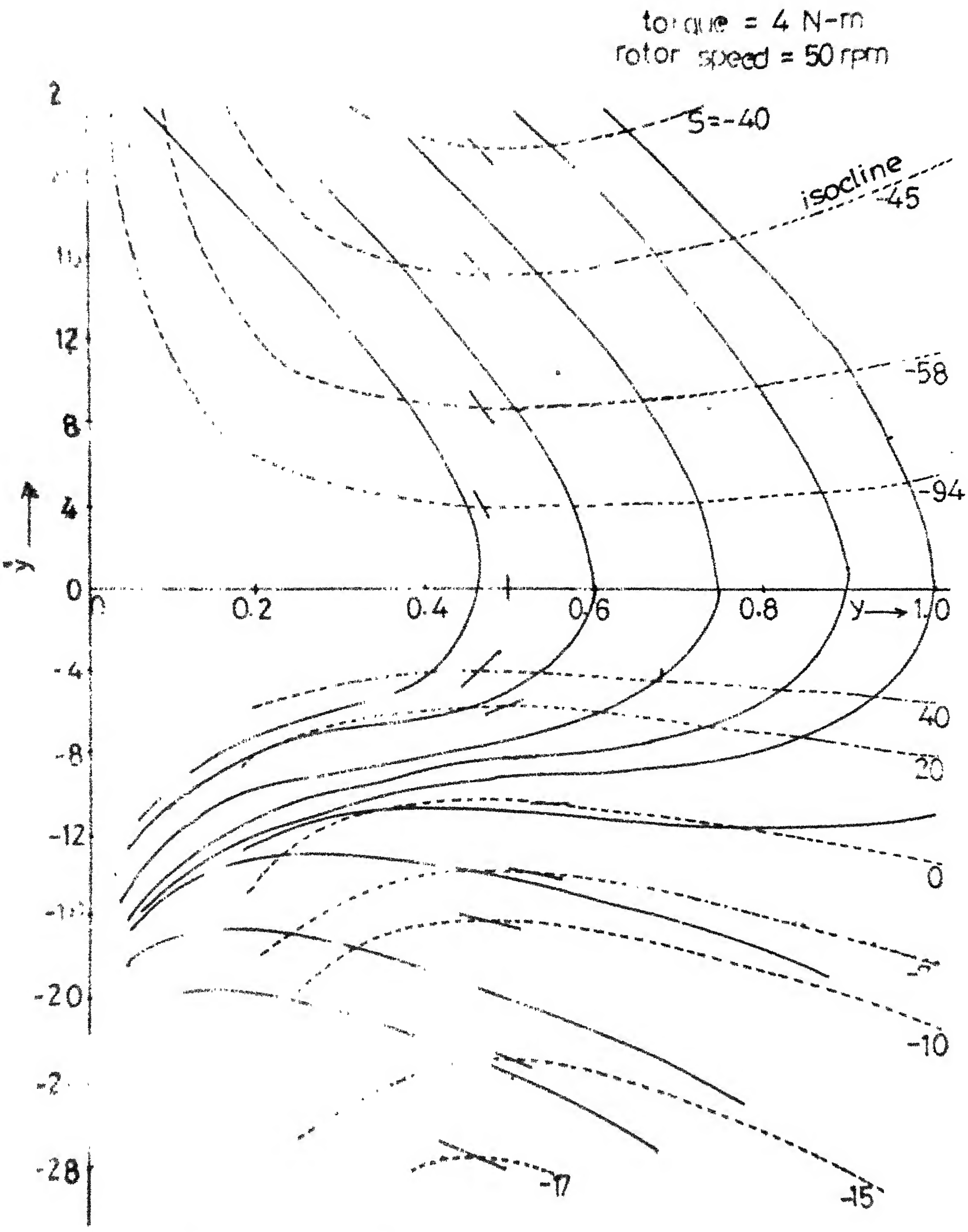


Fig 6.3 : Phase Plane Plot

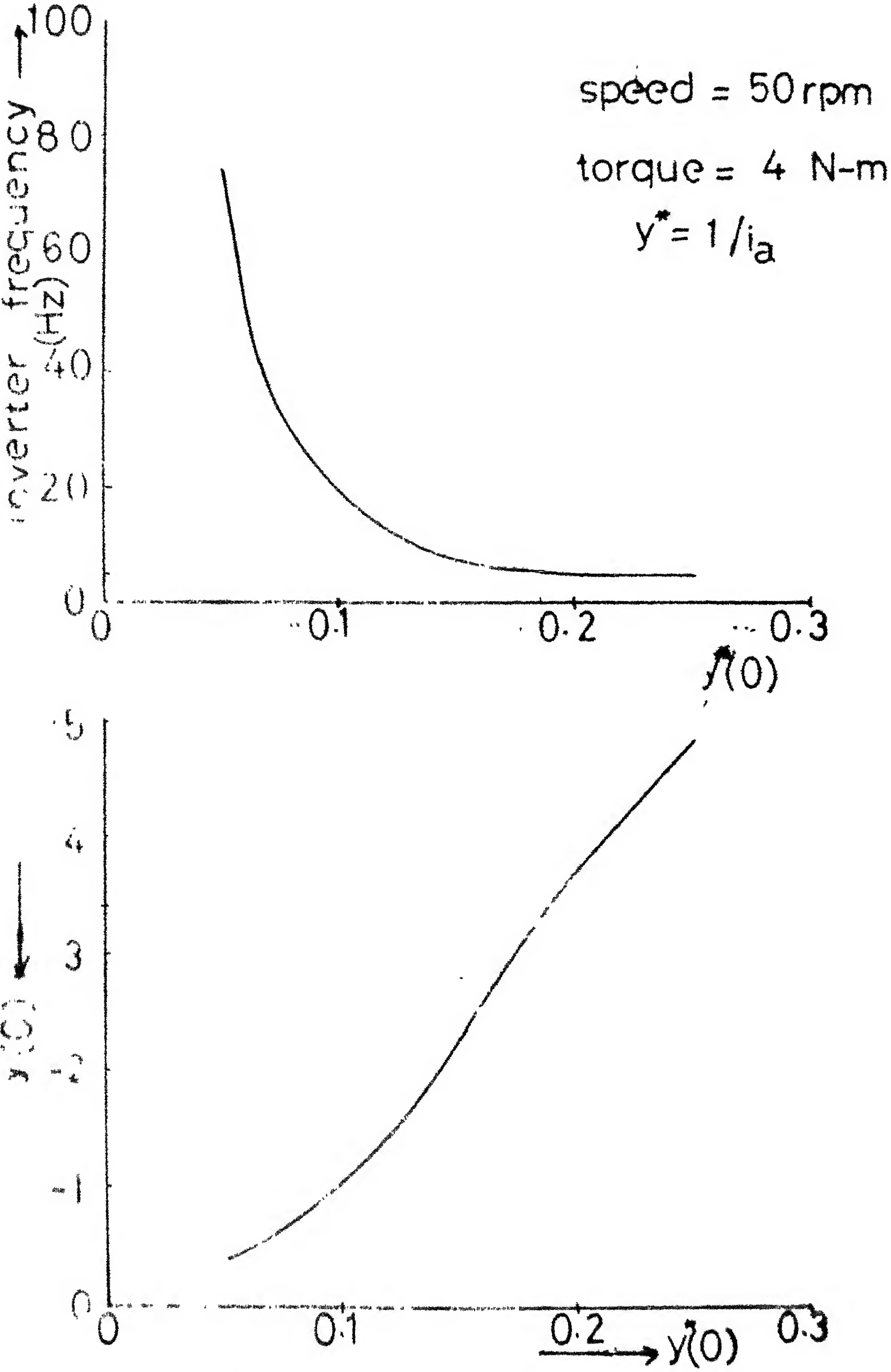
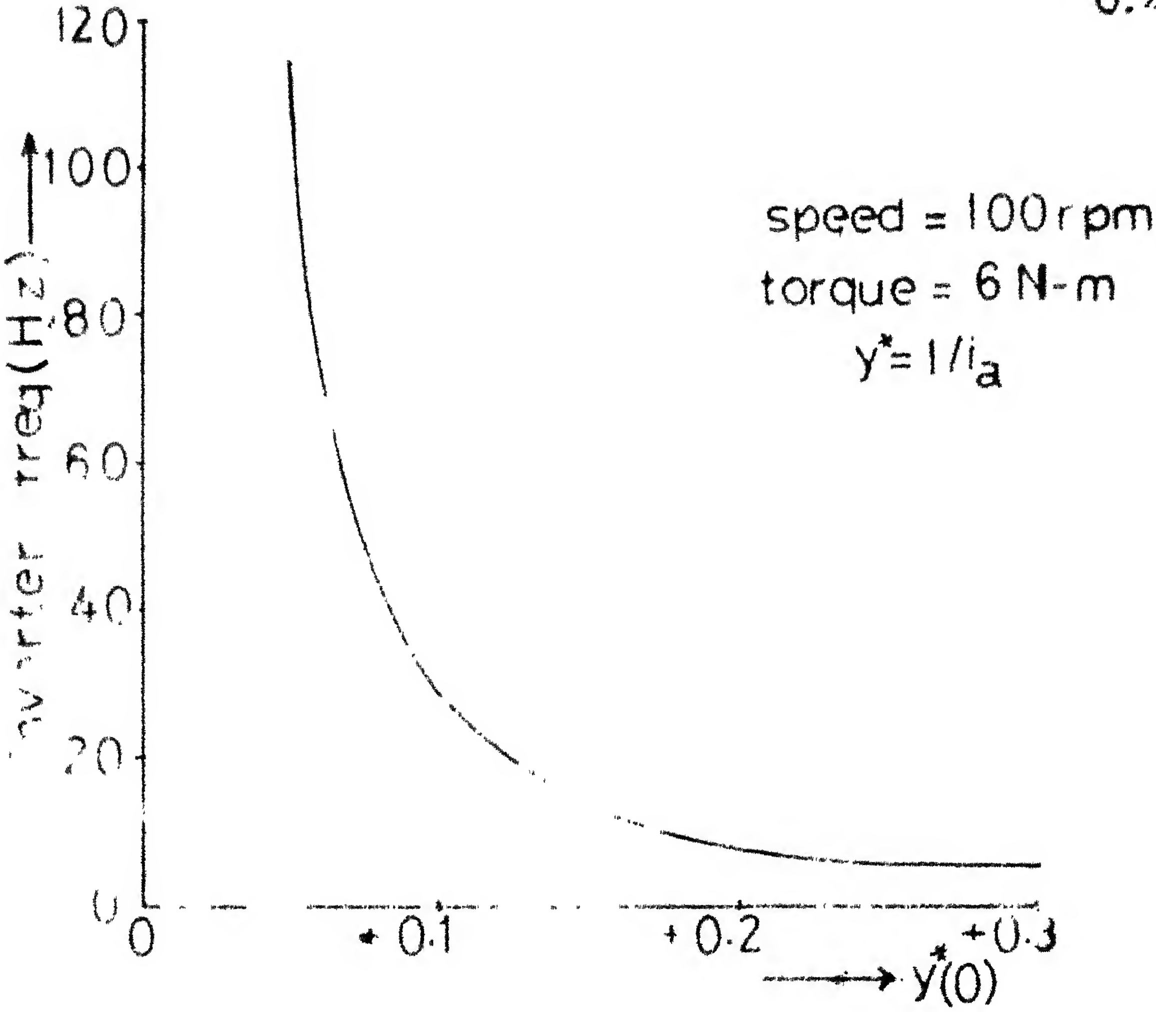


Fig 6.4: Initial Point Of Solution



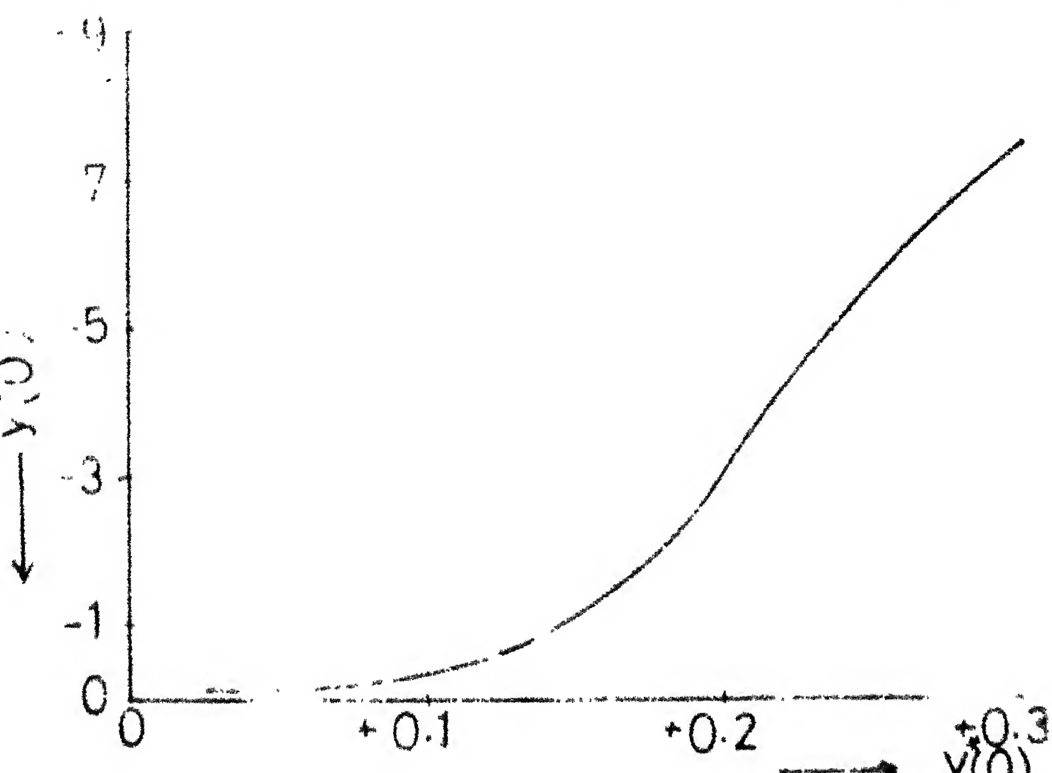


Fig 6.5: Initial Point Of The Solution

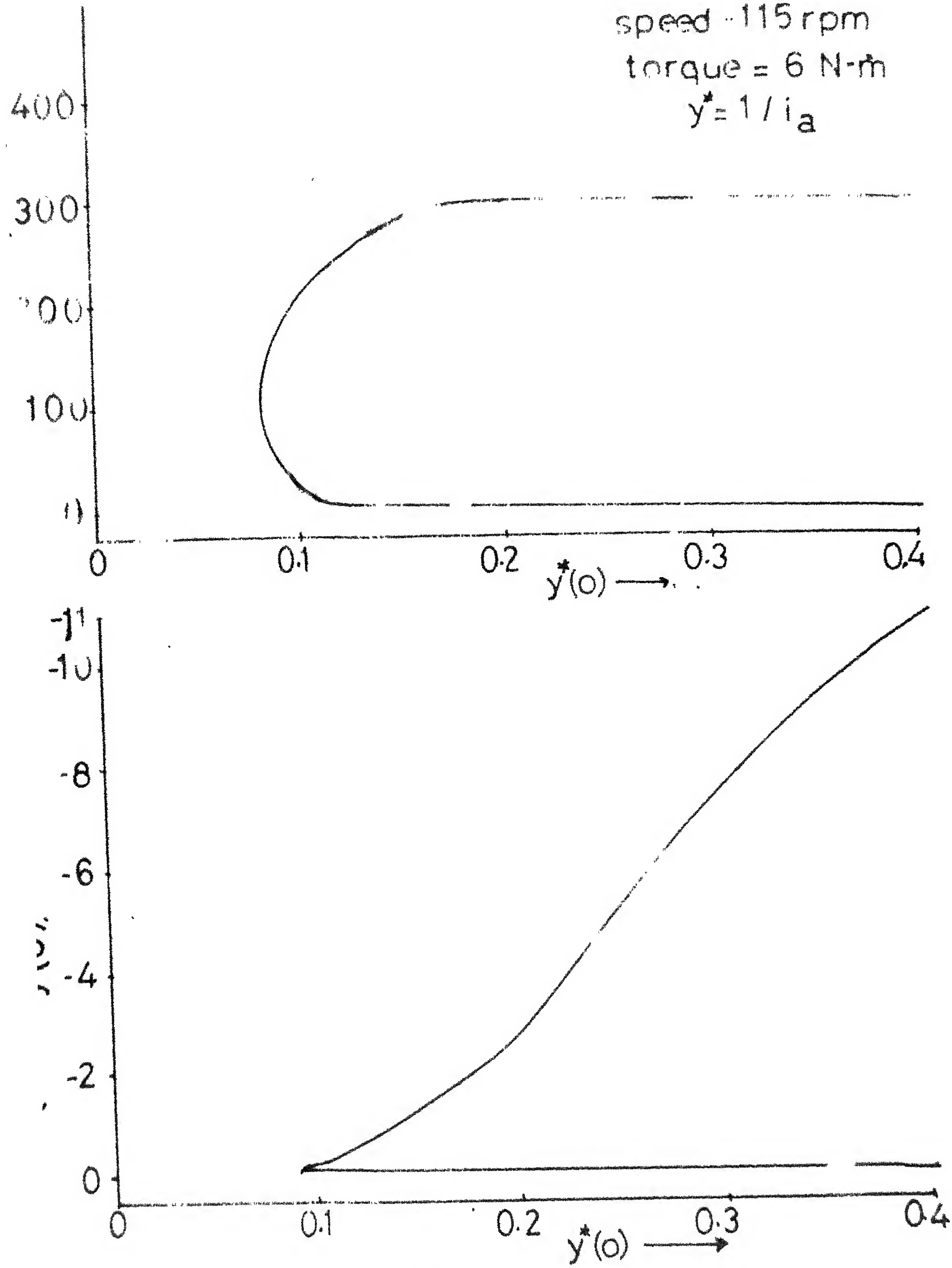
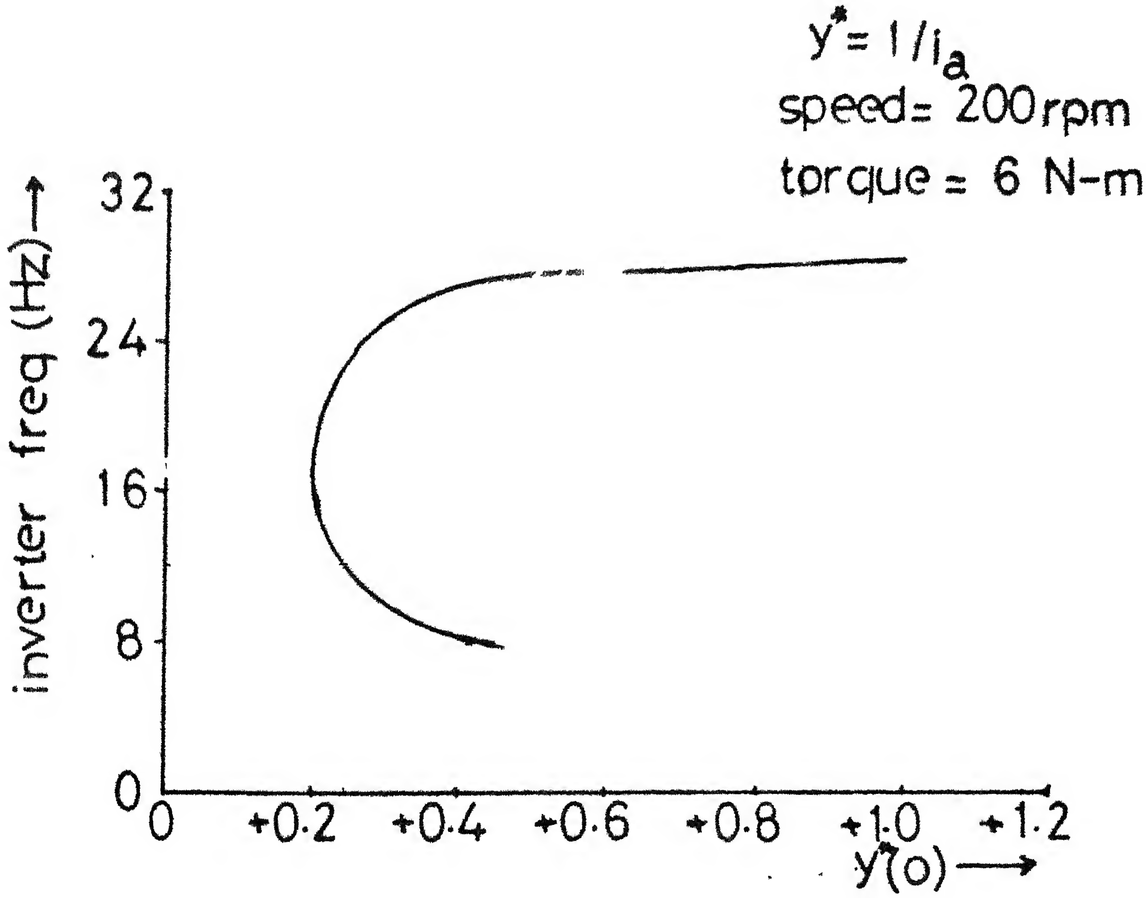
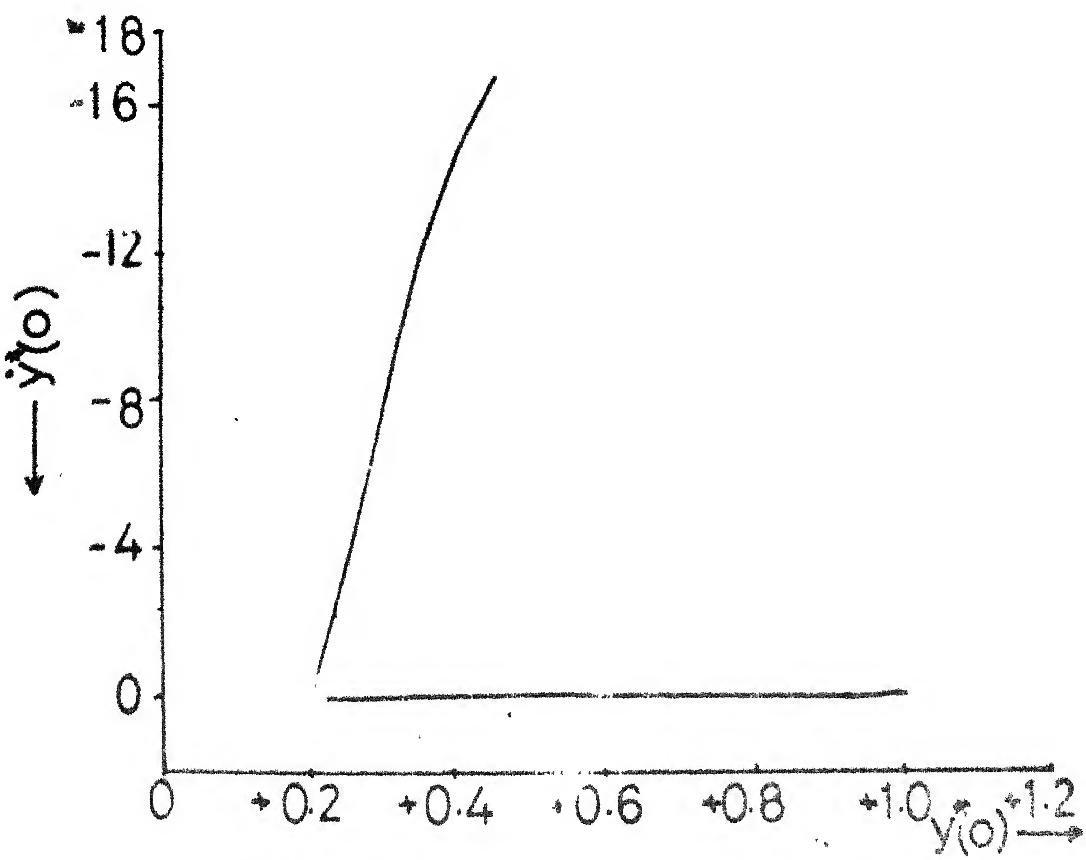
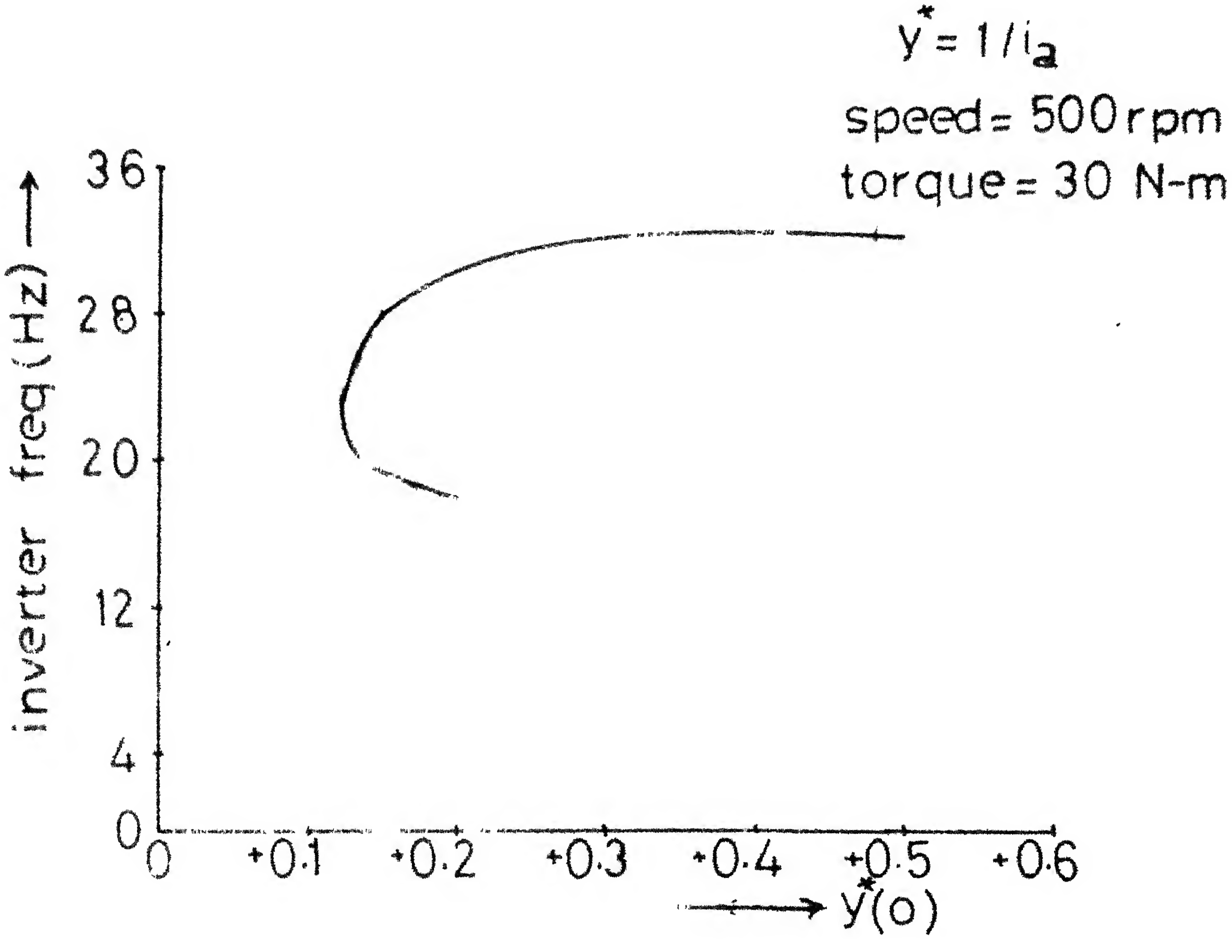


Fig 6.6: Initial Point Of The Solution


y* = 1/ia
speed = 200 rpm
torque = 6 N-m
inverter freq (Hz)
32
24
16
8
0
0
+0.2
+0.4
+0.6
+0.8
+1.0
+1.2
y*(o)


18
-16
-12
-8
-4
0
ẏ*(o)
0
+0.2
+0.4
+0.6
+0.8
+1.0
+1.2
y*(o)



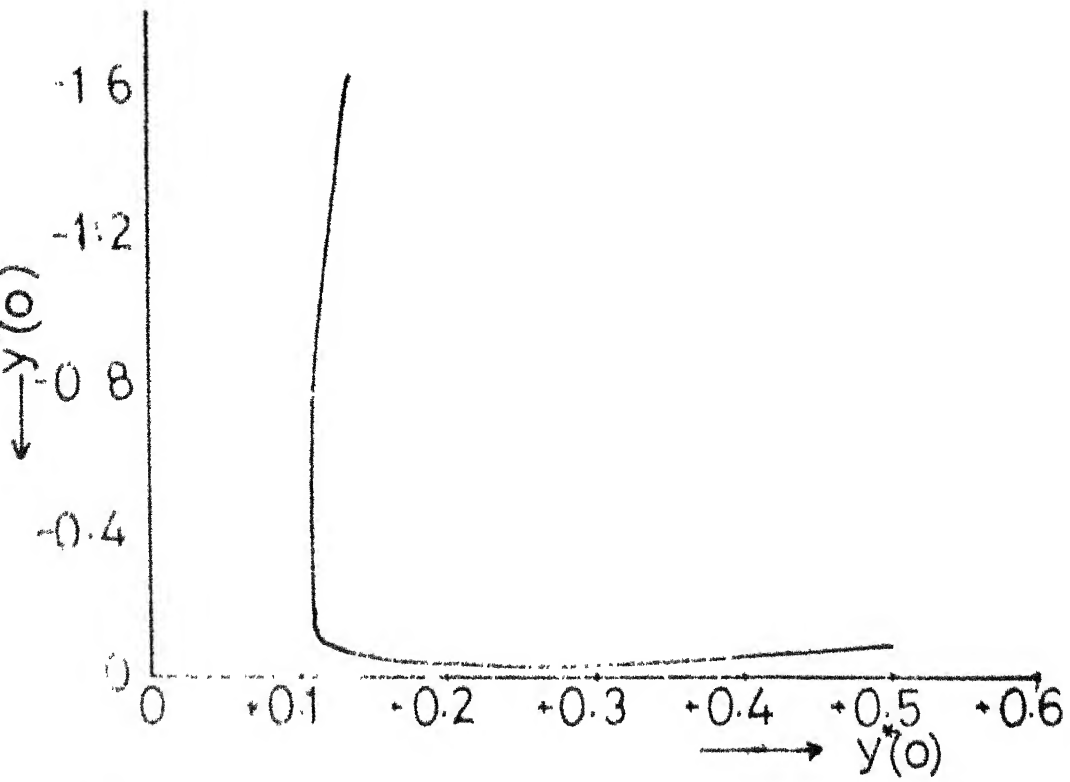


Fig 6.8: Initial Point Of The Solution

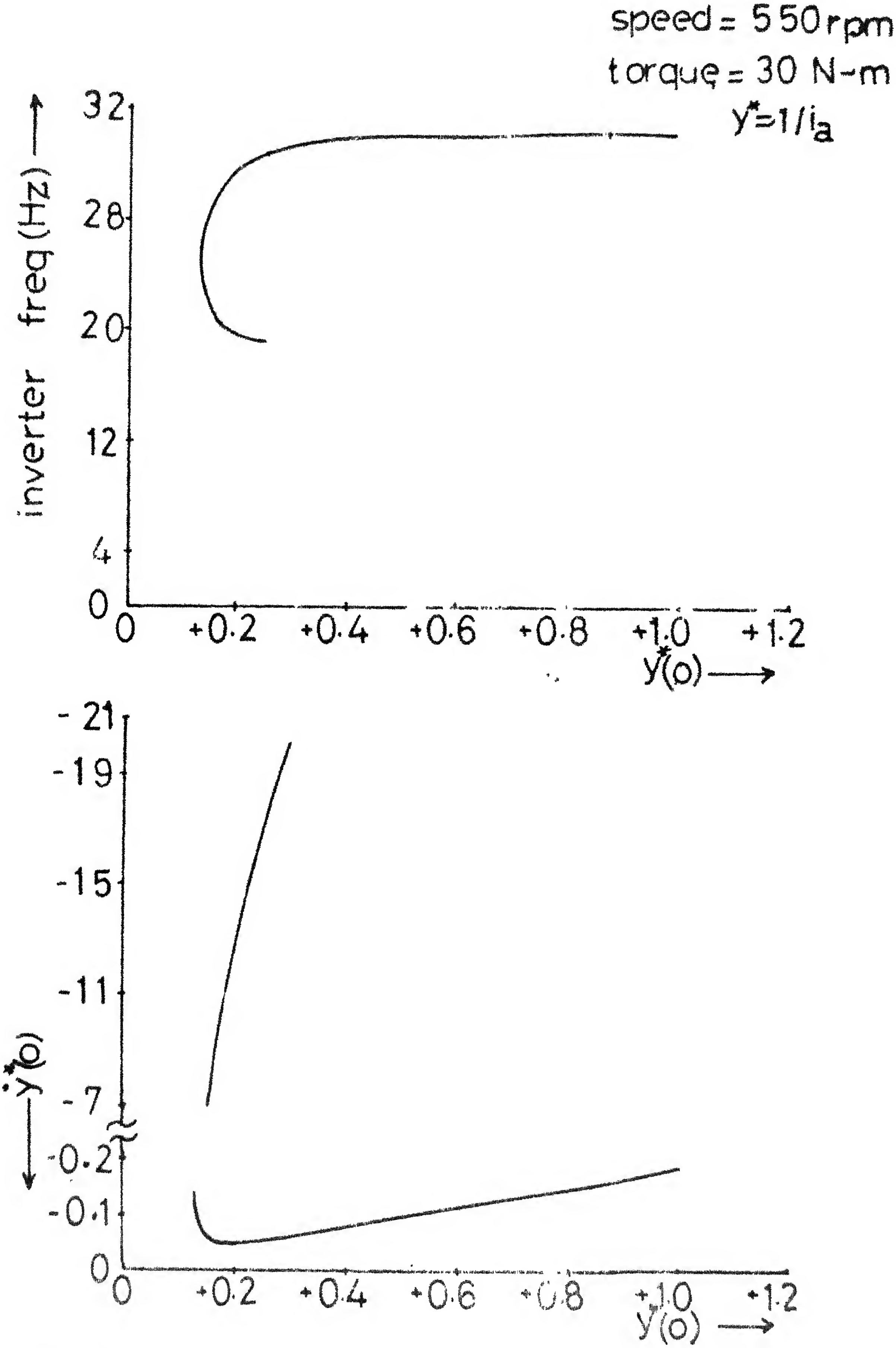
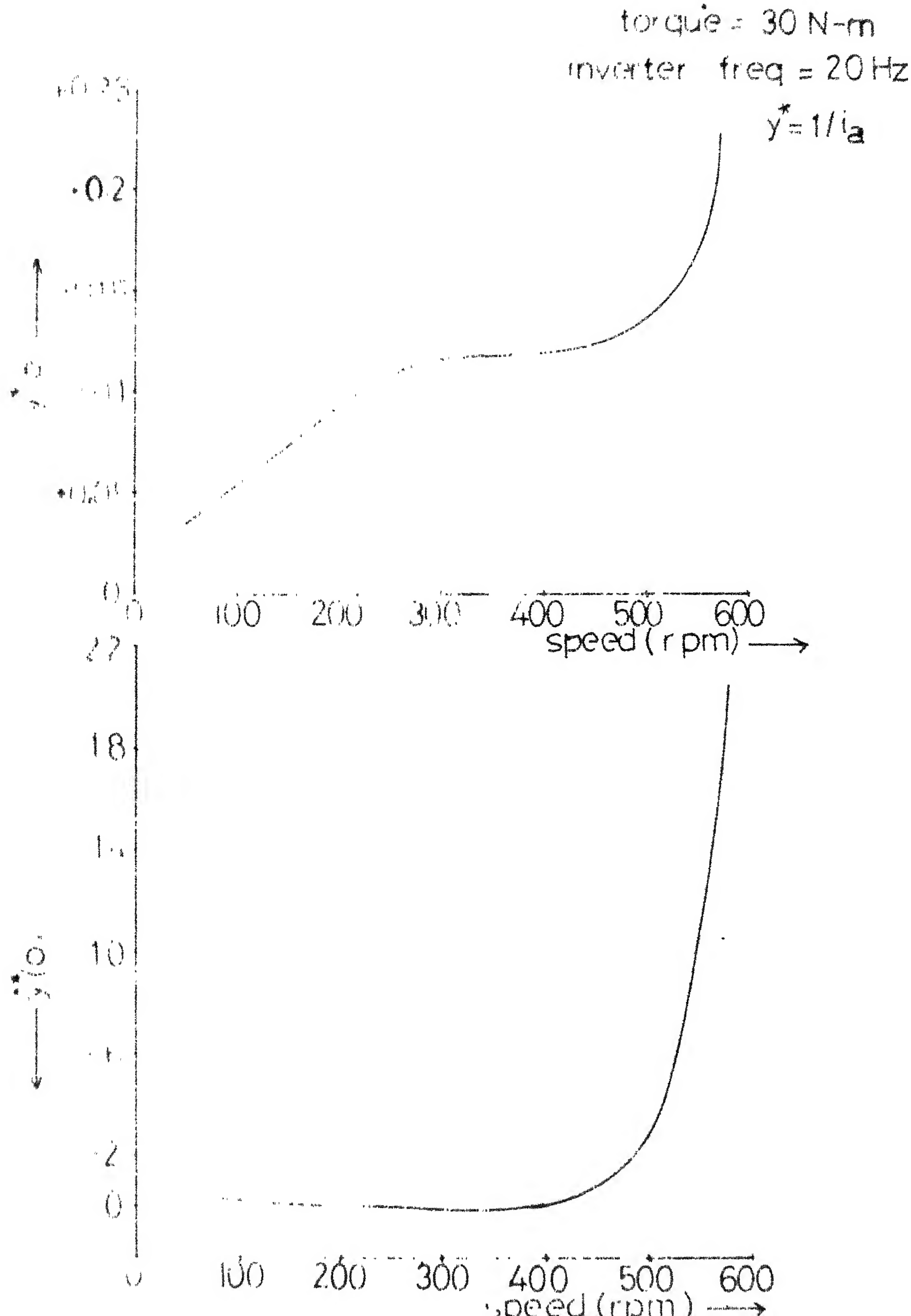


Fig 69: Initial Point Of The Solution


torque = 30 N-m
inverter freq = 20 Hz
y* = 1/i_a
·0.2
0
100
200
300
400
500
600
speed (rpm)
18
10
2
0
100
200
300
400
500
600

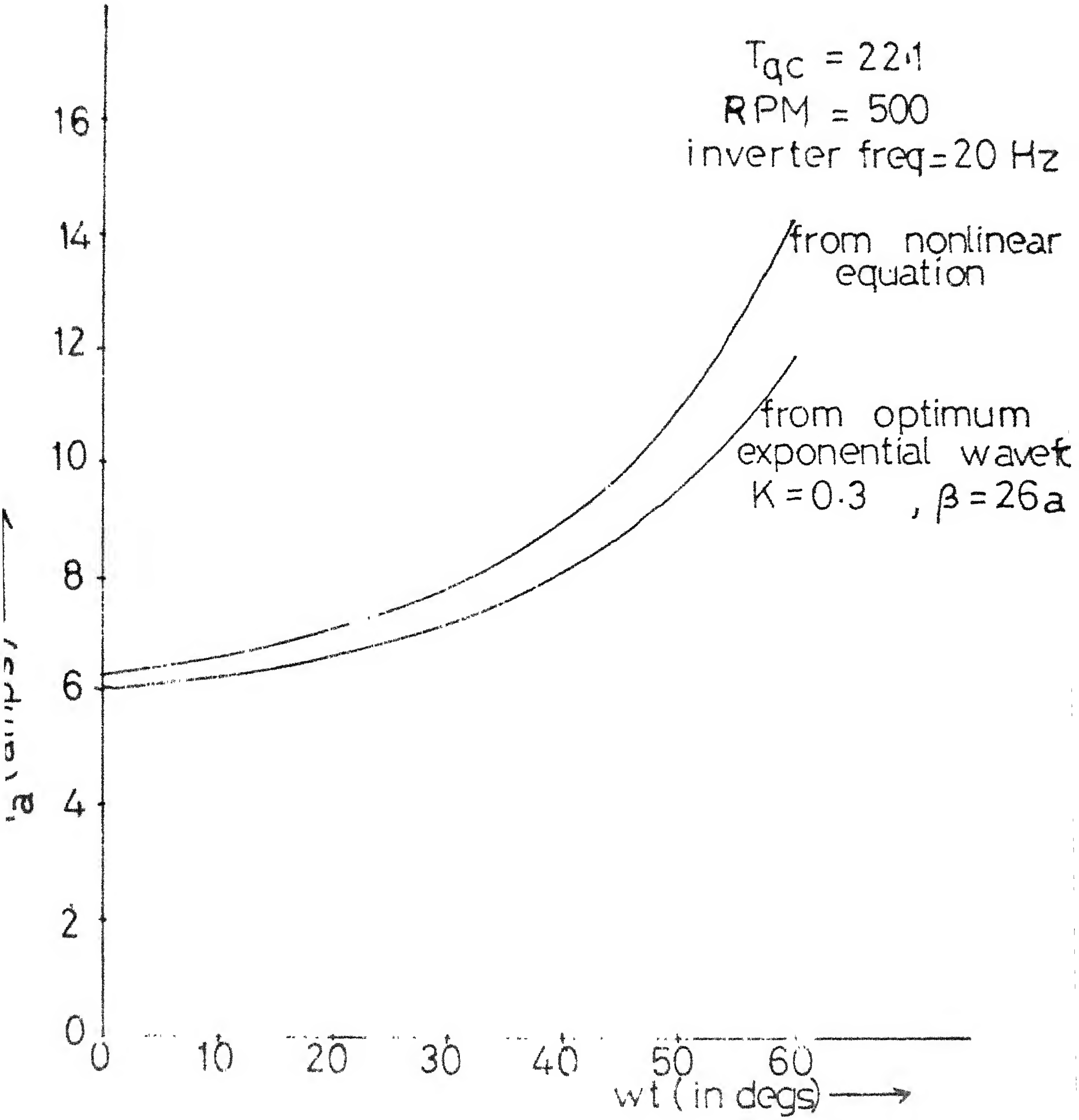


Fig 6.11 : Current of 'a' phase to get constant torqu via nonlinear eq. & min. sixth harmonic torque via exponential modulation

CHAPTER 7

CONCLUSIONS

The analytical expressions for the referred rotor currents of the induction motor fed by ideal square wave current have been obtained. The rotor current has been computed by solving motor performance equations in dq frame. They are also computing by calculating the transfer function of the motor. It has been shown that both these methods give the same results.

The analytical expressions for the induction motor being fed by stator current with cosinusoidal rise and fall during commutation have also been obtained. A more complex time domain expressions are given. It has been shown that as $\omega_c \rightarrow \infty$, this solution approach the solution obtained for square wave current.

The nature of the electromagnetic torque produced by the motor has been studied. The general expression for the torque in a reference frame revolving at arbitary speed has been given. It has been shown that the torque is independent of the speed of the frame. It has been shown that the torque harmonic have frequencies of the type $3p\omega$, where ω is the frequency of stator current and p is a integer. For the case of motor being fed by an inverter, the torque frequencies are of the type $6p\omega$.

The methods to compute electromagnetic torque have been given. This methods are applicable to both ideal and nonideal current source inverter. This torque has harmonics (Fig. 4.5). It has been shown that for computation of a torque harmonic of the order '6p', it is sufficient to consider the current harmonics of the order (6p-1) and (6p+1), (Table 4.2). Detailed calculation show that interaction of fundamental in rotor and these two harmonics in stator contribute the major percentage of the torque.

The control of the torque harmonics by modulation of the dc input current has been studied in Chapters 5 and 6. It is shown that for the balanced system the modulating frequency should be six times the inverter frequency. The cases with exponential and cosinusoidal modulations have been studied to obtain constant torque. The study with exponential modulation shows :

(i) It is possible to obtain a constant torque for stationary rotor with exponential modulation (Table 5.1).

(ii) It is not possible to obtain constant torque for rotating rotor with exponential modulation.

(iii) At speeds less then synchronous speed, the exponential modulation always improves the performance. At speeds greater then synchronous speed, there is deterioration in the performance with exponential modulation (Figs. 5.2 and 5.3).

(iv) At a given rotor speed the parameters of the exponentia modulation have a optimum value. With these values the sixth harmonic torque can be reduced to nearly z. zero (Fig. 5.4). The optimum choice varies with rotor speed (Fig. 5.5).

With cosinusoidal mbdulation,also,it is not possible to get constant torque. Tables 5.2 to 5.6 show that

(i) There is a distinct phase angle for every choice of amplitude and frequency of the cosinusoidal modulating waveform which gives maximum performance index (T_{av}/T_6)

(ii) $\omega_m = 6\omega$, it is possible to reduce sixth harmonic torque nearly equal to zero (Table 5.2).

(iii) There is not much variation in the average torque with $\omega_m = 6\omega$, 12ω or 18ω. With $\omega_m = 3\omega$ there is a significant variation in the average torque.

(iv) For $\omega_m = 3\omega$ or 6ω, the4e is an optimum amplitude for maximum performance index. For case of $\omega_m = 12\omega$ or 18ω, this optimum nature is not observed.

(v) There is not much improvement in performance index with $\omega_m = 12\omega$ or 18ω as compared to $\omega_n = 6\omega$ or 3ω,

(vi) It is possible to reduce 12th harmonic of the torque by current with $\omega_m = 6\omega$ (Table 5.3),

(vii) It can be said that in case the cosinusoidal modulation is being used, the modulating waveform frequency $\omega_m = 6\omega$ gives best control.

The inverse problem of computing the profile of the inverter output current to obtaine the desired torque waveform has also been studied. The stator current for a desired torque waveform is given by the solution of a nonlinear second order equation. The expression for current to obtain a desired torque woveform at stationary rotor has been obtained. If the desired torque waveform is always positive, then the possible shapes of the stator current to obtain this torque is given by Fig. 6.1. The solution of the second order equation need two boundary value conditions. These have been obtained and these turn out to be independent of torque. This second order equation is reduced to two first order equations. These can be solved by phase plane analysis or numerical integration. It can be said that given the rotor speed and constant torque value it is possible to obtain a current waveshape which will give this performance. There is a range of inverter frequency in which this solution is possible (Fig. 6.4 to Fig. 6.9). If the inverter frequency is defined in this range then there is a unique current waveform. Outside this range, there is no solution. The solution of the nonlinear equation gives current which is close to the exponential waveform (Fig. 6.11).

APPENDIX A

Details of the slip ring induction motor used

Primary	Secondary
400 Volts	145 Volts
5.0 Amperes	10 Amperes

3.0 H.P.

50 Cycles

1400 rpm

The parameters of the motor are

Stator leakage inductance	L_{11}	= 0.2453 H
Rotor leakage inductance referred to the stator	L_{22}	= 0.2453 H
Mutual inductance between stator and rotor phases referred to stator	M	= 0.2364 H
Stator resistance		= 1.6 Ohms
Rotor resistance referred to stator		= 3.31 Ohms

```
PROGRAM TO COMPUTE TIME DOMAIN SOLUTION FOR ROTOR
CURRENT FOR STATOR CURRENT WITH COSINOSOIDAL RISE
 AND FALL DURING COMMUTATION

       IMPLICIT REAL(A-Z)
       INTEGER IJ
       DIMENSION X1(1001)
C      THE PARAMETERS ARE TO BE FED INTO THE PROGRAM
       READ(20,*),IDC,F,N,WC
       OPEN(UNIT=21,DEVICE='DSK',FILE='FOR21.DAT')
C      THE FOLLOWING ARE THE MOTOR PARAMETERS
       L1=0.2453;L2=0.2453;M=0.2364;R1=1.6;R2=3.31
       IJ=1
       A=R2/L2
       PI=3.14159
       I=IDC*.66666666
       TC=PI/(2.*WC)
       DELTAT=1/(1000.)
       PIDEL=PI*DELTAT;WR=(4.*PI*N/60.);W=2.*PI*F;T1=1./(6.*F)
       PI3=PI/ 3.0;PI23=PI*2./3.
       T360=PIDEL/W
       TYPE 1050,WR,WC,F,IDC
1050   FORMAT(5X,'WR=',F10.5,5X,'WC=',F10.5,5X,'F=',F10.5,5X,
       1'IDC=',F10.5,/)
       TYPE 1150
1150   FORMAT(18X,'WT',12X,'TIME',11X,'IQ2',11X,'ID2',/)
       K1=M*A/L2;K2=(WR*WR+A*A-WC*WC)/(2.*A)
       K3=1./(WC*WC+K2*K2)
       Y1=EXP(-A*TC)*SIN(WR*TC);Y2=EXP(-A*TC)*COS(WR*TC)
       Y3=EXP(-A*(T1-TC))*SIN(WR*(T1-TC))
       Y4=EXP(-A*(T1-TC))*COS(WR*(T1-TC))
       DO 10 L=0.0,PI,PIDEL
       T=L/W
       IF(0.0.LE.L.AND.L.LT.PI3) GOTO 50
       IF(PI3.LE.L.AND.L.LT.PI23) GOTO 60
       GO TO 70
50     Y11=EXP(-A*T)*SIN(WR*T);Y21=EXP(-A*T)*COS(WR*T)

       IF(T.GT.T360) GO TO 80
C      THE  SUBROUTINE INTIL1 IS CALLED TO CALCULATE THE INTIAL
C      VALUES OF THE CURRENTS IN THE INTERVAL -1
       CALL INTIL1(WR,WC,F,I,A,K1,K2,K3,Y1,Y2,Y3,Y4,K4,K5,K6,C1,C2,C3,
       1C4,C5,C6,Z1,Z2,Z3,Z4,X101,X201)
80     IF(T.GT.TC) GO TO 90
       B11=X101+C1;A11=X201+C2
       X11=A11*Y11+B11*Y21-K4+K5*A*(WC*SIN(WC*T)+K2*COS(WC*T))+K5*WC*
       1(WC*COS(WC*T)-K2*SIN(WC*T))
       IQ2=X11-(M/L2)*(1.732*I*(COS(WC*T)-.5))
       B21=X201+C3;A21=-X101+C4
       X21=A21*Y11+B21*Y21+K6-K5*WR*(WC*SIN(WC*T)+K2*COS(WC*T))
       ID2=X21-(M/L2)*(-1.5*I)
       ID1=-1.5*I;IQ1=1.732*I*(COS(WC*T)-.5)
       X1(IJ)=IQ2+M*IQ1/L2
       IJ=IJ+1
       GO TO 100
90     Y12=EXP(-A*(T-TC))*SIN(WR*(T-TC));Y22=EXP(-A*(T-TC))*COS(WR*
       1(T-TC))
       D11=X201*Y1+X101*Y2+Z1+K4;C11=-X101*Y1+X201*Y2+C5
       X11=C11*Y12+D11*Y22-K4;IQ2=X11-(M/L2)*(-.866*I)
       D21=-X101*Y1+X201*Y2+Z2-K6;C21=-X201*Y1-X101*Y2+C6
       X21=C21*Y12+D21*Y22+K6;ID2=X21-(M/L2)*(-1.5*I)
       ID1=-1.5*I;IQ1=-.866*I
       X1(IJ)=IQ2+M*IQ1/L2
       IJ=IJ+1
100    WRITE(21,*)X1(IJ-1)
```

```
      PROGRAM TO COMPUTE THE REFERED ROTOR CURRENT
      HARMONICS FOR IDEAL SQUARE WAVE STATOR CURRENT
        THROUGH FREQUENCY DOMAIN APPROACH

      COMMON K1,PI,A,CURRT,WR,W,T
      INTEGER S1,TEMP1,TEMP3
      REAL L1,L2,M,K,K1,IDC,PI
      PI = 3.14159
      L1=0.2453;L2=0.2453;M=0.2364;R1=1.6;R2=3.31
      A = R2/L2
      K1 = M*A/L2
      READ(20,*)IDC,F,RPM
      CURRT = (2./3.)*IDC
      WR=RPM*4.*PI/60.0
      W=2.*PI*F
      K = M*1.5*2.
      T=1/F
      WRITE(21,992)RPM,IDC
992   FORMAT(' RPM = ',F8.3,' & IDC = ',F4.2)
      WRITE(21,995)
995   FORMAT(6X,'FREQ.',4X,'X1 MAGNITUDE  ')
      DO 171 II=1,20,2
      S1=II
      CALL HARXS(S1,XMAG,XPH)
      XPH=XPH*180./PI
      WRITE(21,998)S1,XMAG,XPH
998   FORMAT(7X,I2,7X,F8.5,2X,F9.5)
171   CONTINUE
      END
      SUBROUTINE HARCU(FREQN,CUMAG,CUPH)
      COMMON K1,PI,A,CURRT,WR,W,T
      INTEGER FREQN
      FRN =FLOAT( IABS(FREQN))
      FW =(FRN)*W
      AN = (6.*CURRT/T)*(-1.*SIN(FRN*PI/3.)*(2./(FRN*W)))
      CUMAG = (AN)
      CUPH = 0.0
      RETURN; END
      SUBROUTINE HARXS(FREQM,X1MAG,X1PH)
      COMMON K1,PI,A,CURRT,WR,W,T
      INTEGER FREQM
      REAL IM,K1
      FRM = ABS(FLOAT(FREQM))
      FMW = (FRM)*W
      AT = SQRT((-1.*(FMW**2)+A**2+WR**2)**2+(2.*A*FMW)**2)
      ALPHAT = ATAN(2.*A*FMW/(-1.*FMW**2+A**2+WR**2))
      DD = -1.*FMW**2+A**2+WR**2
      IF (DD > 0),GOTO 656
      ALPHAT = PI + ALPHAT
656   CALL HARCU(FREQM,IM,ALPHAM)
      TEMPX1 = K1*A*IM/AT
      IFREQQ=FREQM
      IF ((MOD(FREQM,6)) .EQ. 1) GOTO 67
      FREQM=(-1)*FREQM
      TEMPX1=-1*TEMPX1
67    CONTINUE
      IF (FREQM < 0) GOTO 415
      TEMPX2 = K1*IM*(FMW+WR)/AT
      GOTO 425
415   TEMPX2 = K1*IM*(WR - FMW)/AT
425   TEMPH1=COS(ALPHAM - ALPHAT)
      TEMPH2 = SIN(ALPHAM - ALPHAT)
      TEMPH3 = TEMPX2*TEMPH1 + TEMPX1*TEMPH2
      TEMPH4 = TEMPX1*TEMPH1 - TEMPX2*TEMPH2
      X1PH = ATAN(TEMPH3/TEMPH4)
```

```
      IF (TEMPH4 > 0) GOTO 615
       X1PH = X1PH + PI
615    X1MAG = SQRT(TEMPH3**2+TEMPH4**2)
       FREQM=IFREQ0
       RETURN; END
```

```
          PROGRAM TO COMPUTE TIME DOMAIN SOLUTION OF
          THE ROTOR CURRENT FOR THE CASE OF
          IDEAL SQUARE WAVE STATOR CURRENT

          IMPLICIT REAL(A-Z)
C         THE PARAMETERS ARE TO BE FED INTO THE PROGRAM
          READ(20,*),IDC,F,N
          OPEN(UNIT=21,DEVICE='DSK',FILE='FOR21.DAT')
C         THE FOLLOWING ARE THE MOTOR PARAMETERS
          L1=0.2453;L2=0.2453;M=0.2354;P1=1.0;R2=3.31
          A=R2/L2
          PI=3.14159
          I=IDC*.6666666
          DELTAT=1/(1440.)
          PIDEL=PI*DELTAT;WR=(4.*PI*N/60.);W=2.*PI*F;T1=1./(6.*F)
          PI3=PI/ 3.0;PI23=PI*2./3.
          T360=PIDEL/W
          TYPE 1050,WR,F,IDC
1050      FORMAT(5X,'WR=',F10.5,5X,'F=',F10.5,5X,
          1'IDC=',F10.5,/)
          TYPE 1150
1150      FORMAT(18X,'WT',12X,'TIME',11X,'IO2',11X,'ID2',/)
          K1=M*A/L2
          Y2=1.0;Y1=0.0
          Y3=EXP(-A*(T1))*SIN(WR*(T1))
          Y4=EXP(-A*(T1))*COS(WR*(T1))
          DO 10 L=0.0,PI,PIDEL
          T=L/W
          IF(0.0.LE.L.AND.L.LT.PI3) GOTO 50
          IF(PI3.LE.L.AND.L.LT.PI23) GOTO 60
          GO TO 70
50        Y11=EXP(-A*T)*SIN(WR*T);Y21=EXP(-A*T)*COS(WR*T)
          IF(T.GT.T360) GO TO 80
C         THE  SUBROUTINE INTIL1 IS CALLLED TO CALCULATE THE INTIAL
C         VALUES OF THE CURRENTS IN THE INTERVAL -I
          CALL INTIL1(WR,F,I,A,K1,Y3,Y4,K4,K5,K6,C1,C2,C3,
          1C4,C5,C6,Z1,Z2,Z3,Z4,X101,X201)
80        Y12=EXP(-A*(T))*SIN(WR*(T));Y22=EXP(-A*(T))*COS(WR*
          1(T))
          D11=X101+Z1+K4;C11=X201+C5
          X11=C11*Y12+D11*Y22-K4;IO2=X11-(M/L2)*(-.866*I)
          D21=X201+Z2-K6;C21=-X101+C6
          X21=C21*Y12+D21*Y22+K6;ID2=X21-(M/L2)*(-1.5*I)
          ID1=-1.5*I;IO1=-.866*I
100       X1=IO2+M*IO1/L2
          WRITE(21,*)X1
          GO TO 10
**        FORMAT(10X,3(F10.5,5X))
60        T2=T-(PI3/W)
          Y11=EXP(-A*T2)*SIN(WR*T2);Y21=EXP(-A*T2)*COS(WR*T2)
          IF(T2.GT.T360) GO TO 110
C         NOW THE SUBROUTINE INTIL2 IS CALLED TO CALUCULATE THE INTIAL
C         VALUES OF THE ROTOR CURRENTS IN INTERVAL -II
          CALL INTIL2(WR,F,I,A,K1,Y1,Y2,Y3,Y4,K7,K8,K9,K10,
          1K11,K12,C7,C8,C9,C10,C11,C12,Z5,Z6,Z7,Z8,X102,X202)
110       Y12=EXP(-A*(T2))*SIN(WR*(T2))
          Y22=EXP(-A*(T2))*COS(WR*(T2))
          D32=X202*Y1+X102*Y2+Z5+K7
          C32=X202*Y2-X102*Y1+C11
          X12=C32*Y12+D32*Y22-K7;IO2=X12-(M/L2)*(-1.732*I)
          D42=X202*Y2-X102*Y1+Z6-K10
          C42=-X202*Y1-X102*Y2+C12
          X22=C42*Y12+D42*Y22+K10;ID2=X22-(M/L2)*(0.0)
          ID1=0.0;IO1=-1.732*I
```

```
      X1=IO2+M*IO1/L2
150   WRITE(21,*)X1
      GO TO 10
70    T3=T-(PI23/W)
      Y11=EXP(-A*T3)*SIN(WR*T3);Y21=EXP(-A*T3)*COS(WR*T3)
      IF(T3.GT.T360) GO TO 170
      CALL INTIL3(WR,F,I,A,K1,Y1,Y2,Y3,Y4,K13,K14,K15,K16
     1,K17,C13,C14,C15,C16,C17,C18,Z9,Z10,Z11,Z12,X103,X203)
170   Y12=EXP(-A*(T3))*SIN(WR*(T3))
      Y22=EXP(-A*(T3))*COS(WR*(T3))
      D53=X103*Y2+X203*Y1+Z9-K13
      C53=X203*Y2-X103*Y1+C17
      X13=C53*Y12+D53*Y22+K13;IO2=X13-(M/L2)*(-.866*I)
      D63=-X103*Y1+X203*Y2+Z10-K16
      C63=-X103*Y2-X203*Y1+C18
      X23=C63*Y12+D63*Y22+K16;ID2=X23-(M/L2)*(1.5*I)
      ID1=1.5*I;IO1=-.866*I
      X1=IO2+M*IO1/L2
200   WRITE(21,*)X1
10    CONTINUE
20    CONTINUE
      STOP;END


      SUBROUTINE INTIL1(WR,F,I,A,K1,Y3,Y4,K4,K5,K6,
     1C1,C2,C3,C4,C5,C6,Z1,Z2,Z3,Z4,X101,X201)
C     THIS SUBROUTINE CALCULATES THE INTIAL VALUES OF THE PSUEDO
C     ROTOR CURRENTS IN THE INTERVAL -I

      IMPLICIT REAL(A-Z)
      K4=(.866*K1*I*(1.732*WR+A))/(A*A+WR*WR)
      K5=[illegible]
      K6=(.866*K1*I*(WR-1.732*A))/(A*A+WR*WR)
      C1=[illegible]
      C2=(.866*K1*I+A*C1*(-.866*I*K1*2.))/WR
      Z1=C1-K4
      C3=-K6
      C4=(C3*A-1.5*K1*I)/WR
      Z2=C3+K6
      C5=(-.866*K1*I+K4*A+WR*Z2)/WR
      Z3=C5*Y3+Y4*(Z1+K4)-K4
      C6=(-Z1*WR-1.5*K1*I-K6*A)/WR
      Z4=Y3*C6+Y4*(Z2-K6)+K6
      O11=.5-(Y3);O12=.866-(Y3)
      D=(O11*O11+O12*O12)
      X101=(O11*Z3/D)-(O12*Z4/D)
      X201=(O12*Z3/D)+(O11*Z4/D)
      X11=.5*X101+.866*X201;X21=.5*X201-.866*X101
      RETURN;END


      SUBROUTINE INTIL2(WR,F,I,A,K1,Y1,Y2,Y3,Y4,K7,K8,K9,K10
     1,K11,K12,C7,C8,C9,C10,C11,C12,Z5,Z6,Z7,Z8,X102,X202)
C     THIS PROGRAM CALCULATES THE INTIAL VALUES OF THE PSUEDO ROTOR
C     CURRENTS IN THE INTERVAL--II
      IMPLICIT REAL(A-Z)
      K7=1.732*I*K1*A/(A*A+WR*WR)
      K8=0.0
      K9=0.
      K10=1.732*I*K1*WR/(A*A+WR*WR)
      K11=0.
      K12=0.0
      C7=K7
      C8=(A*C7-.866*K1*I-.866*I*K1)/WR
      Z5=[illegible]*Y1+C7*Y2-K7
      C9=-K10
      C10=(A*C9)/WR
      Z6=C10*Y1+C9*Y2+K10
```

```
      C11=(Z6*WR-1.732*K1*I+K7*A)/WR
      Z7=C11*Y3+Y4*(Z5+K7)-K7
      C12=(-Z5*WR-K10*A)/WR
      Z8=C12*Y3+Y4*(Z6-K10)+K10
      Q11=.5-(Y4*Y2-Y1*Y3);Q12=.866-(Y1*Y4+Y2*Y3)
      D=Q11*Q11+Q12*Q12
      X102=(Q11*Z7/D)-(Q12*Z8/D);X202=(Q12*Z7/D)+(Q11*Z8/D)
      RETURN;END


      SUBROUTINE INTIL3(WR,E,I,A,K1,Y1,Y2,Y3,Y4,K13,K14,
     1K15,K16,K17,C13,C14,C15,C16,C17,C18,Z9,Z10,Z11,Z12,X103,X203)
C     THIS SUBROUTINE CALCULATES THE INTIAL VALUES OF THE PSUEDO
C     ROTOR CURRENTS IN THE INTERVAL--III

      IMPLICIT REAL(A-Z)
      K13=(.866*K1*I*(1.732*WR-A))/(A*A+WR*WR)
      K14=0.0
      K15=0.0
      K16=(.866*K1*I*(WR+1.732*A))/(A*A+WR*WR)
      K17=0.0
      C13=-K13
      C14=(-0.866*K1*I+A*C13)/WR
      Z9=C14*Y1+C13*Y2+K13
      C15=-K16
      C16=(A*C15+(1.732*.866*I*K1C))/WR
      Z10=C16*Y1+C15*Y2+K16
      C17=(Z10*WR-.866*K1*I-K13*A)/WR
      Z11=C17*Y3+Y4*(Z9-K13)+K13
      C18=(-Z9*WR+1.5*K1*I-K16*A)/WR
      Z12=C18*Y3+Y4*(Z10-K16)+K16
      Q11=.5-(Y2*Y4-Y1*Y3);Q12=.866-(Y1*Y4+Y2*Y3)
      D=Q11*Q11+Q12*Q12
      X103=(Q11*Z11/D)-(Q12*Z12/D)
      X203=(Q12*Z11/D)+(Q11*Z12/D)
      RETURN;END
```

```
PROGRAM TO COMPUTE HARMONICS OF ROTOR CURRENT
THROUGH FREQUENCY DOMAIN FOR STATOR CURRENT WITH
COSINUSOIDAL RISE & FALL DURING COMMUTATION

      COMMON K1,PI,A,WC,CURRI,WR,W,T
      INTEGER S1,TEMP1,TEMP3
      REAL L1,L2,M,K,K1,IDC,PI
      PI = 3.14159
      L1=0.2453;L2=0.2453;M=0.2364;R1=1.6;R2=3.31
      A = R2/L2
      K1 = M*A/L2
      READ(20,*)IDC,F,RPM,WC
      CURRI = (2./3.)*IDC
      WR=RPM*4.*PI/60.0
      W=2.*PI*F
      K = M*1.5*2.
      T=1/F
      WRITE(21,992)RPM,IDC
992   FORMAT(' RPM = ',F8.3,' & IDC = ',F4.2)
      WRITE(21,995)
995   FORMAT(5X,'FREQ.',4X,'X1 MAGNITUDE     X1 PHASE(IN DEG.)')
      DO 171 I1=1,29,2
      S1=I1
      CALL HARXS(S1,XMAG,XPH)
      XPH=XPH*180./PI
      WRITE(21,998)S1,XMAG,XPH
998   FORMAT(7X,I2,7X,F8.4,7X,F8.4)
171   CONTINUE
      END
      SUBROUTINE HARXS(FREQM,X1MAG,X1PH)
      COMMON K1,PI,A,WC,CURRI,WR,W,T
      INTEGER FREQM
      REAL IM,K1
      TYPE *,K1,PI,A,WC,CURRI,WR,W,T
      FRM = ABS(FLOAT(FREQM))
      FRW = (FRM)*W
      AT = SQRT((-1.*(FRW**2)+A**2+WR**2)**2+(2.*A*FRW)**2)
      ALPHAT = ATAN(2.*A*FRW/(-1.*FRW**2+A**2+WR**2))
      DD = -1.*FRW**2+A**2+WR**2
      IF (DD > 0) GOTO 656
      ALPHAT = PI + ALPHAT
656   CALL HARCU(FREQM,IM,ALPHAM)
      TEMPX1 = K1*A*IM/AT
      ITFREQ=FREQM
      IF ((MOD(FREQM,3)) .EQ. 1) GOTO 67
      FREQM=(-1)*FREQM
      TEMPX1=TEMPX1*(-1)
67    CONTINUE
      IF (FREQM < 0) GOTO 415
      TEMPX2 = K1*IM*(FRW+WR)/AT
      GOTO 425
415   TEMPX2 = K1*IM*(WR - FRW)/AT
425   TEMPH1=COS(ALPHAM - ALPHAT)
      TEMPH2 = SIN(ALPHAM - ALPHAT)
      TEMPH3 = TEMPX2*TEMPH1 + TEMPX1*TEMPH2
      TEMPH4 = TEMPX1*TEMPH1 - TEMPX2*TEMPH2
      X1PH = ATAN(TEMPH3/TEMPH4)
      IF (TEMPH4 > 0) GOTO 616
      X1PH = X1PH + PI
616   X1MAG = SQRT(TEMPH3**2+TEMPH4**2)
      FREQM=ITFREQ
      RETURN; END
      SUBROUTINE HARCU(FREQM,CUMAG,CUPH)
      COMMON K1,PI,A,WC,CURRI,WR,W,T
      INTEGER FREQM
```

```
      FRN =FLOAT( IABS(FREQN))
      FW =(FRN)*W
      FDIFF=(FW-WC)
      AN = (6.*CURRT/T)*(-1.*SIN(FRN*PI/3.)*((2./(FRN*W))
     1-1./(FW+WC)-1./(FDIFF))+0.5*(1./(FDIFF)
     1-1./(FW+WC))*(COS(FW*PI/(2.*WC)+FRN*PI/3.)
     1+COS((FW*PI/(2.*WC))+2.*FRN*PI/3.)))
      BN = (6.*CURRT/T)*(1./(FDIFF)-1./(FW+WC))*
     1(SIN((FW*PI/(2.*WC))+FRN*PI/3.)
     1 + SIN((FW*PI/(2.*WC))+2.*FRN*PI/3.))
      CURMG = SQRT(AN**2+BN**2)
      CURPH = ATAN(-1.*BN/AN)
      IF (AN>0) GOTO 606
      CURPH = CURPH + PI
606   RETURN;END
```

```
      GO TO 10
**    FORMAT(10X,3(F10.5,5X))
60    T2=T-(PI3/W)
      Y11=EXP(-A*T2)*SIN(WR*T2);Y21=EXP(-A*T2)*COS(WR*T2)
      IF(T2.GT.T360) GO TO 110
C     NOW THE SUBROUTINE INTIL2 IS CALLED TO CALUCULATE THE INITIAL
C     VALUES OF THE ROTOR CURRENTS IN INTERVAL -II
      CALL INTIL2(WR,WC,F,I,A,K1,K2,K3,Y1,Y2,Y3,Y4,K7,K8,K9,K10,
     1K11,K12,C7,C8,C9,C10,C11,C12,Z5,Z6,Z7,Z8,X102,X202)
110   IF(T2.GT.TC) GOTO 120
      B32=X102+C7;A32=X202+C8
      X12=A32*Y11+B32*Y21-K7+K8*(WC*SIN(WC*T2)+K2*COS(WC*T2))+K9*
     1(WC*COS(WC*T2)-K2*SIN(WC*T2))
      IQ2=X12-(M/L2)*(1.732*I*(.5*COS(WC*T2)-1.))
      B42=X202+C9;A42=-X102+C10
      X22=A42*Y11+B42*Y21+K10-K11*(WC*SIN(WC*T2)+K2*COS(WC*T2))-K12*
     1(WC*COS(WC*T2)-K2*SIN(WC*T2))
      ID2=X22-(M/L2)*(-1.5*I*COS(WC*T2))
      ID1=-1.5*I*COS(WC*T2);IQ1=1.732*I*(.5*COS(WC*T2)-1.)
      X1(IJ)=IQ2+M*IQ1/L2
      IJ=IJ+1
      GO TO 150
120   Y12=EXP(-A*(T2-TC))*SIN(WR*(T2-TC))
      Y22=EXP(-A*(T2-TC))*COS(WR*(T2-TC))
      D32=X202*Y1+X102*Y2+Z5+K7
      C32=X202*Y2-X102*Y1+C11
      X12=C32*Y12+D32*Y22-K7;IQ2=X12-(M/L2)*(-1.732*I)
      D42=X202*Y2-X102*Y1+Z6-K10
      C42=-X202*Y1-X102*Y2+C12
      X22=C42*Y12+D42*Y22+K10;ID2=X22-(M/L2)*(0.0)
      ID1=0.0;IQ1=-1.732*I
      X1(IJ)=IQ2+M*IQ1/L2
      IJ=IJ+1
150   WRITE(21,*)X1(IJ-1)
      GO TO 10
70    T3=T-(PI23/W)
      Y11=EXP(-A*T3)*SIN(WR*T3);Y21=EXP(-A*T3)*COS(WR*T3)
      IF(T3.GT.T360) GO TO 170
      CALL INTIL3(WR,WC,F,I,A,K1,K2,K3,Y1,Y2,Y3,Y4,K13,K14,K15,K16
     1,K17,C13,C14,C15,C16,C17,C18,Z9,Z10,Z11,Z12,X103,X203)
170   IF(T3.GT.TC) GOTO 155
      B53=X103+C13;A53=X203+C14
      X13=A53*Y11+B53*Y21+K13-K14*(WC*SIN(WC*T3)+K2*COS(WC*T3))-K15*
     1(WC*COS(WC*T3)-K2*SIN(WC*T3))
C     NOW THE SUBROUTINE INTIL3 IS CALLED TO CALCULATE THE INITIAL
C     VALUES OF THE ROTOR CURRENTS IN THE INTERVAL-III
      IQ2=X13-(M/L2)*(-.866*I*(1.+COS(WC*T3)))
      B63=X203+C15;A63=-X103+C16
      X23=A63*Y11+B63*Y21+K16+K17*(WC*SIN(WC*T3)+K2*COS(WC*T3))-
     11.732*K15*(WC*COS(WC*T3)-K2*SIN(WC*T3))
      ID2=X23-(M/L2)*(1.5*I*(1.-COS(WC*T3)))
      ID1=1.5*I*(1.0-COS(WC*T3));IQ1=-.866*I*(1.+COS(WC*T3))
      X1(IJ)=IQ2+M*IQ1/L2
      IJ=IJ+1
      GO TO 200
155   Y12=EXP(-A*(T3-TC))*SIN(WR*(T3-TC))
      Y22=EXP(-A*(T3-TC))*COS(WR*(T3-TC))
      D53=X103*Y2+X203*Y1+Z9-K13
      C53=X203*Y2-X103*Y1+C17
      X13=C53*Y12+D53*Y22+K13;IQ2=X13-(M/L2)*(-.866*I)
      D63=-X103*Y1+X203*Y2+Z10-K16
      C63=-X103*Y2-X203*Y1+C18
      X23=C63*Y12+D63*Y22+K16;ID2=X23-(M/L2)*(1.5*I)
      ID1=1.5*I;IQ1=-.866*I
      X1(IJ)=IQ2+M*IQ1/L2
      IJ=IJ+1
200   WRITE(21,*)X1(IJ-1)
10    CONTINUE
20    CONTINUE
      STOP;END
```

```
      SUBROUTINE INTIL1(WR,WC,F,I,A,K1,K2,K3,Y1,Y2,Y3,Y4,K4,K5,K6,
     1C1,C2,C3,C4,C5,C6,Z1,Z2,Z3,Z4,X101,X201)
C     THIS SUBROUTINE CALCULATES THE INTIAL VALUES OF THE PSUEDO
C     ROTOR CURRENTS IN THE INTERVAL -I

      IMPLICIT REAL(A-Z)
      K4=(.866*K1*I*(1.732*WR+A))/(A*A+WR*WR)
      K5=.866*K1*I*K3/A
      K6=(.866*K1*I*(WR-1.732*A))/(A*A+WR*WR)
      C1=K4-K5*K2*A-K5*WC*WC
      C2=(.866*K1*I+A*C1-K5*A*WC*WC+K5*K2*WC*WC)/WR
      Z1=C2*Y1+C1*Y2-K4+K5*A*WC-K5*WC*K2
      C3=K5*K2*WR-K6
      C4=(C3*A-1.5*K1*I+K5*WR*WC*WC)/WR
      Z2=C4*Y1+C3*Y2+K6-K5*WR*WC
      C5=(-.866*K1*I+K4*A+WR*Z2)/WR
      Z3=C5*Y3+Y4*(Z1+K4)-K4
      C6=(-Z1*WR-1.5*K1*I-K6*A)/WR
      Z4=Y3*C6+Y4*(Z2-K6)+K6
      O11=.5-(Y2*Y4-Y1*Y3);O12=.866-(Y4*Y1+Y2*Y3)
      D=(O11*O11+O12*O12)
      X101=(O11*Z3/D)-(O12*Z4/D)
      X201=(O12*Z3/D)+(O11*Z4/D)
      X11=.5*X101+.866*X201;X21=.5*X201-.866*X101
      RETURN;END


      SUBROUTINE INTIL2(WR,WC,F,I,A,K1,K2,K3,Y1,Y2,Y3,Y4,K7,K8,K9,K10,
     1,K11,K12,C7,C8,C9,C10,C11,C12,Z5,Z6,Z7,Z8,X102,X202)
C     THIS PROGRAM CALCULATES THE INTIAL VALUES OF THE PSUEDO ROTOR
C     CURRENTS IN THE INTERVAL--II
      IMPLICIT REAL(A-Z)
      K7=1.732*I*K1*A/(A*A+WR*WR)
      K8=((.866*I*K1*A-1.5*I*K1*WR)*K3)/(2.*A)
      K9=.133*I*K1*WC*K3/A
      K10=1.732*I*K1*WR/(A*A+WR*WR)
      K11=((1.5*K1*I*A+.866*K1*WR*I)*K3)/(2.*A)
      K12=.75*I*K1*WC*K3/A
      C7=K7-K8*K2-K9*WC
      C8=(A*C7-.866*K1*I-K8*WC*WC+K9*K2*WC)/WR
      Z5=C8*Y1+C7*Y2-K7+K8*WC-K9*K2
      C9=-K10+K11*K2+K12*WC
      C10=(-1.5*K1*I+A*C9+K11*WC*WC-K12*K2*WC)/WR
      Z6=C10*Y1+C9*Y2+K10-K11*WC+K12*K2
      C11=(Z6*WR-1.732*K1*I+K7*A)/WR
      Z7=C11*Y3+Y4*(Z5+K7)-K7
      C12=(-Z5*WR-K10*A)/WR
      Z8=C12*Y3+Y4*(Z6-K10)+K10
      O11=.5-(Y4*Y2-X1*Y3);O12=.866-(Y1*Y4+Y2*Y3)
      D=O11*O11+O12*O12
      X102=(O11*Z7/D)-(O12*Z8/D);X202=(O12*Z7/D)+(O11*Z8/D)
      RETURN,END


      SUBROUTINE INTIL3(WR,WC,F,I,A,K1,K2,K3,Y1,Y2,Y3,Y4,K13,K14,
     1K15,K16,K17,C13,C14,C15,C16,C17,C18,Z9,Z10,Z11,Z12,X103,X203)
C     THIS SUBROUTINE CALCULATES THE INTIAL VALUES OF THE PSUEDO
C     ROTOR CURRENTS IN THE INTERVAL--III

      IMPLICIT REAL(A-Z)
      K13=(.866*K1*I*(1.732*WR-A))/(A*A+WR*WR)
      K14=((.866*K1*I*A+1.5*K1*I*WR)*K3)/(2.*A)
      K15=.133*K1*I*WC*K3/A
      K16=(.866*K1*I*(WR+1.732*A))/(A*A+WR*WR)
      K17=((.866*K1*WR*I-1.5*K1*I*A)*K3)/(2.*A)
      C13=-K13+K14*K2+K15*WC
      C14=(-1.732*K1*I+A*C13+K14*WC*WC-K15*K2*WC)/WR
      Z9=C14*Y1+C13*Y2+K13-K14*WC+K15*K2
      C15=-K16-K17*K2+1.732*K15*WC
```

```
C16=(A*C15-K17*WC*WC-1.732*K15*K2*WC)/WR
Z10=C16*Y1+C15*Y2+K16+K17*WC+1.732*K15*K2
C17=(Z10*WR-.866*K1*I-K13*A)/WR
Z11=C17*Y3+Y4*(Z9-K13)+K13
C18=(-Z9*WR+1.5*K1*I-K16*A)/WR
Z12=C18*Y3+Y4*(Z10-K16)+K16
Q11=.5-(Y2*Y4-Y1*Y3);Q12=.866-(Y1*Y4+Y2*Y3)
D=Q11*Q11+Q12*Q12
X103=(Q11*Z11/D)-(Q12*Z12/D)
X203=(Q12*Z11/D)+(Q11*Z12/D)
RETURN;END
```

PROGRAM TO OBTAIN FOURIER SERIES COMPONENTS OF A PERIODIC WAVEFORM WITH SAMPLED POINTS GIVEN FOR A PERIOD
(This program needs 721 points.This number can be changed)

```
      DIMENSION TPTS(721)
      PI=3.14159
      PIDEL=PI/(720.)
      DO 100 I=1,721
      READ(20,*),TPTS(I)
100   CONTINUE
      DO 300 IFREQ=1,20,2
      AN=0.0
      BN=0.0
      DO 200 I=1,720
      PITIM1=(FLOAT(I)-1.)*PIDEL
      PITIM2=(FLOAT(I))*PIDEL
      AN=AN+((TPTS(I)*COS(FLOAT(IFREQ)*PITIM1)
     1+TPTS(I+1)*COS(FLOAT(IFREQ)*PITIM2))/2.)
      BN=BN+((TPTS(I)*SIN(FLOAT(IFREQ)*PITIM1)
     1+TPTS(I+1)*SIN(FLOAT(IFREQ)*PITIM2))/2.)
200   CONTINUE
      AN=AN*PIDEL*2./PI
      BN=BN*PIDEL*2./PI
      HARMAG=SQRT(AN**2+BN**2)
      HARPH=ATAN(AN/BN)
      IF (BN > 0)  GOTO 679
      HARPH=HARPH+PI
679   HARPH=HARPH*180/PI
      WRITE(21,999)IFREQ,HARMAG,HARPH
999   FORMAT('FOR FREQ',I3,'HAR. IS',F9.5,2X,F9.5)
300   CONTINUE
      END
```

PROGRAM TO COMPUTE TORQUE HARMONICS . THE STATOR CURRENT IS TO BE GIVEN IN LINE 35&38. (Here the case of square wave current is taken)

```
      INTEGER DIFVAR,S1,S2,TEMP1,TEMP3,FCOMP,FLIM
      REAL L1,L2,M,K,K1,IDC,PI,IMAG,IPH,L
      LOGICAL DONREV,DONPLS
      DIMENSION CURID1(721),CURIQ1(721)
      COMMON WR,K1,PI,A,CURRI,W,T,CURID1
      PI = 3.14159
      L1=0.2453;L2=0.2453;M=0.2364;R1=1.6;R2=3.31
      A = R2/L2
      K1 = M*A/L2
      FLIM = 8
      READ(20,*)IDC,F,RPM
      CURRI = (2./3.)*IDC
      W=2.*PI*F
      K = M*1.5*2.
      T=1/F
      FCOMP = 6
      PI3=PI/3.
      PI23=PI*2./3.
      PIDEL=PI/720.
      [illegible]*1.*PI/60.0
      [illegible]=[illegible]*2./3.
      [illegible]=A*([illegible])
      DO 601 II=1,721
      G=(FLOAT(II)-1.)*PIDEL
      [illegible]=G/[illegible]
      IF (0.LT.II.AND.II.LE.240) GOTO 650
      IF (240.LT.II.AND.II.LE.480) GOTO 660
      GOTO 670
650   CURID1(II)=ID1
      GOTO 601
660   [illegible]-(PI3/[illegible])
      CURID1(II)=ID1
      GOTO 601
670   CURID1(II)=0.0
601   CONTINUE
      CALL [illegible](TORQ)
      [illegible]=TORQ*180./PI
104   [illegible] = 0.0
      [illegible] = 0.0
      [illegible] = .TRUE.
      IF ((MOD(FCOMP,12)) .EQ. 0) GOTO 101
      DONPLS = .TRUE.
      GOTO 102
101   DONPLS = .FALSE.
102   DIFVAR = 6
      CALL SPLIT(FCOMP,S1,S2)
120   CALL HARCU(S1,IMAG,IPH)
      CALL HARXS(S2,XMAG,XPH)
      [illegible] = K*IMAG*XMAG
      IF (S1 < 0) GOTO 305
      IF (S2 < 0) GOTO 315
      [illegible] = IPH - XPH
      IF ((S1) > (S2)) GOTO 103
      [illegible] = -1.*[illegible]
      [illegible] = -1.*[illegible]
      GOTO 103
315   [illegible] = IPH + XPH
      GOTO 103
305   IF (S2 < 0) GOTO 355
      [illegible] = -1.*[illegible]
```

```
      GOTO 315
355   IF ((S2) < (S1)) GOTO 335
      TTOMAG = -1.*TTOMAG
      TTOPH = IPH - XPH
      GOTO 103
335   TTOPH = XPH -IPH
103   CALL VTOADD(TORPH,TORMAG,TTOMAG,TTOPH)
      DONREV = .NOT.(DONREV)
      IF (DONREV) GOTO 130
      TEMP1 = S1
      S1 = S2
      S2 = TEMP1
      GOTO 120
130   DONPLS = .NOT.(DONPLS)
      IF (DONPLS) GOTO 140
      S1 = S1+DIFVAR
      S2 = S2 + DIFVAR
      GOTO 125
140   S1 = S1 - DIFVAR
      S2 = S2 - DIFVAR
125   DIFVAR = DIFVAR + .6
      IF (DIFVAR>37) GOTO 180
      GOTO 120
180   TRATIO=TORM/TORMAG
      TYPE *,IA,VARM,TRATIO
      WRITE(31,998)RPM,IDC,VARM,IA,TORM,FCOMP,TORMAG,TRATIO
998   FORMAT(3X,F8.3,8X,F6.3,10X,F6.3,13X,I2,
     110X,F10.4,3X,I2,6X,F10.4,8X,F8.4)
      GOTO 170
170   CONTINUE
      END
      SUBROUTINE SPLIT(FREOCO,HALF1,HALF2)
C     THIS SUBROUTINE GIVES STARTING NUMBERS
      INTEGER HALF1,HALF2,FREOCO
      HALF1 = FREOCO/2
      HALF2 = HALF1
100   HALF1 = HALF1 - 1
      HALF2 = HALF2 + 1
      IF ((MOD(HALF1,3)) .EQ. 0) GOTO 100
      IF ((MOD(HALF1,2)) .EQ. 0) GOTO 100
      IF ((MOD(HALF1,6)) .EQ. 5) GOTO 110
      HALF2 = HALF2*(-1)
      GOTO 121
110   HALF1 = HALF1*(-1)
121   CONTINUE
      TYPE *, HALF1,HALF2
      RETURN; END
      SUBROUTINE HARXS(FREOM,X1MAG,X1PH)
      DIMENSION CURID1(721)
      COMMON WR,K1,PI,A,CURRI,W,T,CURID1
      INTEGER FREOM
      READ IM,K1
      FRM = ABS(FLOAT(FREOM))
      FMW = (FRM)*W
      AT = SQRT((-1*(FMW**2)+A**2+WR**2)**2+(2.*A*FMW)**2)
      ALPHAT = ATAN(2.*A*FMW/(-1.*FMW**2+A**2+WR**2))
      DD = -1.*FMW**2+A**2+WR**2
      IF (DD > 0) GOTO 656
      ALPHAT = PI + ALPHAT
656   CALL HARCI(FREOM,IM,ALPHAM)
      TEMPX1 = K1*A*IM/AT
      IF (FREOM < 0) GOTO 415
      TEMPX2 = K1*IM*(FMW+WR)/AT
      GOTO 425
415   TEMPX2 = K1*IM*(FMW-WR)/AT
425   TEMPH1=COS(ALPHAM - ALPHAT)
      TEMPH2 = SIN(ALPHAM - ALPHAT)
      TEMPH3 = TEMPX2*TEMPH1 + TEMPX1*TEMPH2
      TEMPH4 = TEMPX1*TEMPH1 - TEMPX2*TEMPH2
      X1PH = ATAN(TEMPH3/TEMPH4)
      IF (TEMPH4 > 0) GOTO 616
      X1PH = X1PH + PI
616   X1MAG = SQRT(TEMPH3**2+TEMPH4**2)
      RETURN; END
      SUBROUTINE VTOADD(T1PH,T1MAG,T2MAG,T2PH)
```

```
         PI = 3.14159
         TT01 = (T1MAG*COS(T1PH))+(T2MAG*COS(T2PH))
         TT02 = (T1MAG*SIN(T1PH))+(T2MAG*SIN(T2PH))
         T1MAG = SQRT(TT01**2+TT02**2)
         T1PH = ATAN(TT02/TT01)
         IF (TT01 > 0) GOTO 626
         T1PH = PI + T1PH
626      RETURN;END
         SUBROUTINE HARCU(IFREQN,CUMAG,CURPH)
         DIMENSION CURID1(721)
         COMMON WR,K1,PI,A,CURRI,W,T,CURID1
         PIDEL=PI/(720.)
         IFREQ=IABS(IFREQN)
         AN=0.0
         BN=0.0
         DO 703 I=1,720
         PITIM1=(FLOAT(I)-1.)*PIDEL
         PITIM2=(FLOAT(I))*PIDEL
         AN=AN+((CURID1(I)*COS(FLOAT(IFREQ)*PITIM1)
         1+CURID1(I+1)*COS(FLOAT(IFREQ)*PITIM2))/2.)
         BN=BN+((CURID1(I)*SIN(FLOAT(IFREQ)*PITIM1)
         1+CURID1(I+1)*SIN(FLOAT(IFREQ)*PITIM2))/2.)
703      CONTINUE
         AN=AN*PIDEL*2./PI
         BN=BN*PIDEL*2./PI
         CUMAG=SQRT(AN**2+BN**2)
         CURPH=ATAN(-1.*BN/AN)
         IF (AN>0.0) GOTO 399
         CURPH=CURPH+PI
399      CONTINUE
         END
         SUBROUTINE AVTOR(TORMAG)
         REAL M,K,K1
         COMMON WR,K1,PI,A,CURRI,W,T,CURID1
         M=0.2364
         K=M*1.5*2.
         TORMAG=0.0
         DO 963 III=1,12,2
         II=III
         IF ((MOD(II,3) .EQ. 0)) GOTO 963
         CALL HARCU(II,CUMAG,CUPH)
         CALL HARKS(II,XMAG,XPH)
         TTOROM=K*CUMAG*XMAG*SIN(CUPH-XPH)
         TORMAG=TORMAG+TTOROM
963      CONTINUE
         RETURN
         END
```

```
      PROGRAM TO SOLVE FIRST ORDER NONLINEAR EQUATION
      THIS ALSO COMPUTES THE INVERTER FREQUENCY

      REAL K1,L2,MM
       OPEN(UNIT=21,DEVICE='DSK',FILE='FOR21.DAT')
       R2=3.31
       L2=0.2453
       A=R2/L2
       RPM=500.
       WR=4.00*3.14*RPM/60.00
       MM=0.2364
       Z=MM*1.50*2.00
       K1=MM*A/L2
       TO=22.1402
       ALPHA=-4.00*Z* K1*WR/(3.00*TO)
       BETA=SQRT(A**2+WR**2)
       AV=0.50*(1+1.732*A/WR)
       BV=-1.732*WR*(1.00+A**2/WR**2)/2.00
       CV=1.732*0.50/WR
       DO  674  II=31190,31195,7
       Y=0.157
       X=-1.*FLOAT(II)/10000.
       TYPE *,II
       XEND=(AV*X+CV*Y)
       AINMNT=(XEND-X)/500.00
       YD=(AV*Y+BV*X)
       WRITE(21,*)X,Y
       TTEMP=0.0
       DO 754 I=1,500
       YD=(ALPHA/X-BETA**2*X)/Y-2.*A
       YOLD=Y
       Y=YD*AINMNT+Y
       TTEMP=(2/(Y+YOLD))*AINMNT+TTEMP
       X=X+AINMNT
       WRITE(5,*)X,YD
       TIME=TTEMP
       FREQ=1/(1.*2.*TIME)
       WRITE(5,*)XEND,FREQ
       WRITE(21,*)XEND,Y
       WRITE(21,*)FREQ,YD
*      X=X+TTEMP
*10    CONTINUE
674    CONTINUE
       STOP
       END
```

REFERENCES

[1] W. Farrer and J.D. Miskin, 'Quasi-sinewave fully regenerative inverter', Proc. IEE, vol. 120, No.9, pp 969-976, Sept. 1973.

[2] P.C. Krause and C.H. Thomas, 'Simulation of symmetrical induction machinery', IEEE Trans., PAS, vol. PAS-84, No.11, pp 1035-1053, Nov. 1965.

[3] P.C. Krause, 'Method of multiple reference frames applied to analysis of symmetrical induction machinery', IEEE Trans. PAS, vol. PAS-87, pp 218-227, Jan. 1968.

[4] T.A. Lipo and E.P. Cornell, 'State variable steady state analysis of controlled current induction motor drive', IEEE Trans., Ind. Appl., vol. IA-11, pp 704-712, Nov/Dec. 1975.

[5] I. Yasuoka and Y. Tsunehiro, 'Estimation of commutating capacitance of current impressed inverter used induction motor drive', Elect.Engg. in Japan, vol. 94, No.4,1974.

[6] V.V. Sastri, 'Current source inverter fed induction motor analysis using boundary value approach', IEEE Trans., IECI, pp 1978, 1977.

[7] S. Yuvarajan, B. Ramaswamy and V. Subramanya, 'Analysis of current controlled inverter fed induction motor using digital simulation', IEEE Trans. IECI, vol.27,No.2, May 1980.

[8] T.H. Chin and H. Tomita, 'The principles of eliminating pulsating torque in current source inverter-induction motor system', Conf. record of IEEE 1977 Intel Semiconductor Power Conv. Conf., Pg 910.

[9] T.H. Chin, 'A new controlled current type inverter with improved performance', Conf. record of IEEE 1977 Intel Semiconductor Power Conv.Conf., pp 185.

[10] A. Nabae, 'Pulse amplitude modulated current source inverter for a.c. drives', Conf. record of IAS 1978, pp 539.

[11] G.K. Geighton, 'Current source inverter fed induction motor torque pulsation', Proc. IEE, vol. 127, Pt.B, No.4, pp 231-239, July 1980.

[12] S.S. Kumar, 'Steady state analysis of three phase current source inverter and current fed induction motor', M.Tech. thesis, Elect.Engg.Deptt., IIT Kanpur, July 1983.

[13] R. Clay, 'Nonlinear Networks and Systems', Wiley Interscience Book.